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AUTHORITY

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Active Control

For Aircraft Landing Gear

Thesis

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1stLt. USAF

SCHOOL OF ENGINEERING

WRIGHT-PATTERSON AIR FORCE BASE, OHIO

76

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Active Control

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THESIS

Presented to the Faculty of the School of Engineering

Of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master Of Science

by

Ronald A. De Yoe, B.S.E.E.
1stLt. USAF

Graduate Electrical Engineering

June 1970

Preface

This study investigates the use of active control for aircraft landing gear to minimize runway imposed vibration. The optimal control approach is taken, with both the (1-cos wt) bump and runway spectral density considered for inputs to a two degree of freedom linear landing gear model. The conjugate gradient numerical technique is used for problem solution.

I wish to extend my sincerest appreciation to Mr. Ronald O. Anderson and Captain James Dillow of the Control Criteria Branch, Air Force Flight Dynamics Laboratory for making the completion of this study possible. I wish to thank also Major John C. Schoop, my faculty adviser, for his interest and helpful suggestions.

Finally, I wish to thank my wife and children for accepting a part-time husband and father for the duration of this study.

Ronald A. De Yoe

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List of Symbols

Cs	Suspension damping constant
Fs	Force produced by actuator
F1	Aircraft weight on landing gear
F2	Unsprung weight of landing gear
g	Gravity equal to 32 ft/sec^2
Ks	Suspension spring constant
Kt	Linearized tire spring constant
M1	F1/g
M2	F2/g
P	Weighting factor
Q1	Ks/F1
Q2	Cs/F2
Q3	Ksg(F1+F2)/F1F2
Q4	Csg(F1+F2)/F1F2
Q5	Ktg/F2
Q6	Ksg/F2
Q7	Csg/F2
R	M1/M2

ABSTRACT

The purpose of this thesis is to investigate the use of active control on aircraft landing gear. The problem of vibration isolation is considered using the optimal control approach to establish the control force necessary to minimize runway imposed vibration. A two degree of freedom linear landing gear model is considered, and both the (1-cos wt) bump and an equivalent deterministic input derived from runway spectra, are used as vertical forcing function inputs to the model. An integral square cost function is formed with the integrand consisting of the absolute acceleration of the aircraft squared, plus a weighting factor times the actuator control force squared. The conjugate gradient technique is used for numerical solution of the problem.

Results indicate that considerable improvement in vibration isolation could be gained with the active control landing gear system over the present passive systems for most paved runway surfaces. It is recommended that further study be made with a nonlinear landing gear model and also with the relative displacement of the model added to the cost formulation.

Active Control For
Aircraft Landing Gear

I. Introduction

Background

Whenever aircraft make contact with a runway, whether ~~it~~ be taxiing, take-off or landing, they are subjected to many structural stresses and strains. These stresses and strains on the airframe are caused by forces transmitted through the shock strut or landing gear by imperfections in the runway surface. The large, heavy, flexible, airplanes, such as the C141, B52, and C-5A, in particular present unique problems of structural fatigue because of their combination of increased size and increased structual flexibility. When a B52 moves over a runway the wings move up and down in an oscillatory motion due to the bumps and indentations in the runway surface, and although less noticeable to the observer, the entire airframe of the airplane is subjected to these same vibrations from runway roughness or uneveness.

Problem

Most aircraft landing gear or shock struts use what is commonly known as an "oleo" to isolate the aircraft from runway imposed shock and vibration. The oleo can be considered a passive spring and dashpot system. By

passive, it is meant that the shock strut simply "reacts" to any shock or vibration input from the runway, and it is not subject to any outside or external control. The oleo is also primarily designed to absorb landing impact energy and herein lies a problem. The shock from landing impact and the vibration from runway roughness are two distinct environments. Landing impact energy can be much greater in magnitude than runway vibration energy; thus, if emphasis in design of the oleo is placed on absorption of landing impact energy, the oleo is then too hard of a suspension system to efficiently absorb vibration energy from runway roughness. In addition, the input to the shock strut from landing impact is a well defined deterministic input, while the vibrational input from runway uneveness varies with different surfaces and is random in nature. An optimum passive system can be designed for absorption of landing impact energy, but it is more difficult to design such a system to absorb random runway vibration energy (Ref 3:9).

A number of studies such as those of Ref(1-5) have shown that considerable improvement in performance can be gained by using an active shock and vibration isolation system,the improvement in performance being less vibration transmitted to the isolated body by the active suspension system with the same amount of suspension clearance or "rattle space" as that of the passive

suspension system. In terms of the aircraft, this would mean less vibration transmitted to the airframe with the active system given the same suspension clearance as that of the passive shock strut system or oleo.

The active suspension system is a feedback control system. With the active system, sensing devices would be used to give a preview reading of the change in runway height from some mean value. Other parameters such as aircraft speed, weight, wing lift etc. would be then fed to an on-board computer. The computer would produce shock strut control signals which minimize induced loads and vibration from runway uneveness.

When the sensing system felt or spotted a depression in the runway, the shock strut would be extended just the right amount to maintain the one "g" force or aircraft's weight at all times. This would tend to hold the airplane at its initial level. Similarly when the airplane passes over a bump, the shock strut would be retracted the proper amount to again maintain the one "g" force and hold the airplane at its initial level.

Purpose and Scope

The purpose of this study is to investigate the use of active control on aircraft landing gear. The objective is to find the control force which when applied to the shock strut or landing gear, will minimize runway imposed vibration. It is necessary to first establish the nature

of the control force in order to determine if such an actuator whether hydraulic, pneumatic, or whatever could do the job. Secondly, it is desired to investigate the optimal control approach to the problem with the integral square value of absolute acceleration and control as a cost criterion. The numerical solution will be obtained using the conjugate gradient technique developed by Lasden, Mitter and Warren (Ref 8).

II. Dynamic Model

The dynamic model to use for this kind of study is not readily apparent. The choice of exact dynamics of a landing gear of a particular airplane does not lend itself toward a feasible study at this time. This is partly due to the fact that there are so many undefined variables such as what kind of actuator, how it is to be used and any limitations resulting from a particular application of such an actuator. It therefore seems reasonable to keep the model simple and general enough to be able to draw some meaningful conclusions. The one reservation that must be kept in mind is that if the general model restricts the number of degrees of freedom or makes assumptions about the environment, then there is the possibility that the optimum solution for the general model may be less than optimum for a particular application.

The active system chosen to represent the landing gear is the two degree of freedom model shown in Figure 1. At first, the study began with a model exactly like that of Figure 1 except it had no passive spring element Ks or damper Cs. This meant the only suspension element was the active device such as a hydraulic or pneumatic actuator. But in examining this kind of system more closely it can be seen that for isolation of the aircraft from runway roughness, the actuator would have to

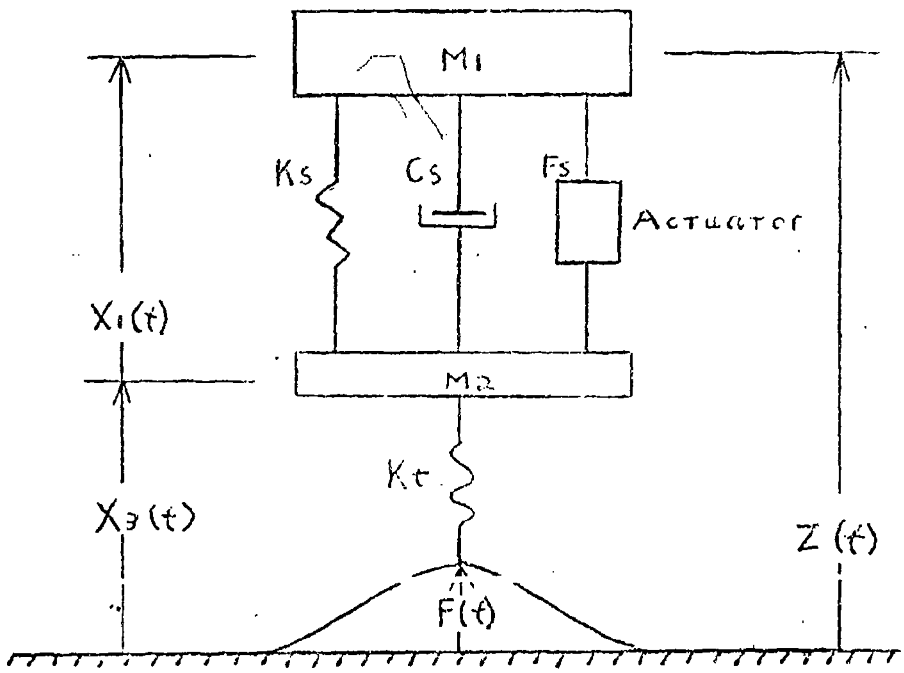


Figure 1

Dynamic Model

supply a constant "F1" force under variable displacement required by the runway bumps and indentations. The state of the art of this kind of actuator is not advanced enough to do the job. However, by adding the spring Ks and damper Cs as in Figure 1, the problem takes on a completely different form. The actuator must supply only the additional force required together with the spring and damper to maintain the F1 force on the airplane.

Referring to Figure 1, the aircraft is the rigid mass M1. The landing gear suspension is idealized as a massless element providing forces between M1 and the unsprung mass M2. The total suspension force is then the sum of the forces produced by the spring Ks, the damper Cs, and the actuator Fs. M2 is supported by the linear tire spring Kt, and the system is excited by the bump F(t). The displacement of M1 measured relative to the fixed frame of reference, is denoted by Z. The displacement of M1 relative to M2 is denoted by X1, and the displacement of M2 relative to the ground or fixed reference is denoted by X3.

<u>System Equations</u>

The equations of motion for the suspension system are:

$$M_1 \ddot{Z} = F_s - K_s X_1 - C_s \dot{X}_1 - M_1 g \tag{1}$$

$$M_2 \ddot{X}_3 = -F_s + K_s X_1 + C_s \dot{X}_1 + K_t (F(t) - X_3) - M_2 g \tag{2}$$

with the Kinematic conditions

$$\begin{aligned} Z &= X_1 + X_3 \\ \dot{Z} &= \dot{X}_1 + \dot{X}_3 \\ \ddot{Z} &= \ddot{X}_1 + \ddot{X}_3 \end{aligned} \tag{3}$$

Assumptions

First, the runway whether it be concrete or soil is assumed to be rigid, and the landing gear wheel is restrained to follow the runway profile (no wheel hop is permissible). Secondly, the model is a rigid mass model and will predict only low frequency behavior, but the frequencies of interest are fairly low and are in the range of .5 to 32 Hz. This frequency range represents many of the important natural modes of oscillation of aircraft such as the B52, and C-5A. The third assumption is of course for the active landing gear system in this study, it is assumed that the characteristics of the bump on the runway are known before the landing gear wheel rides over it. In other words, some kind of a runway preview scanning system is used. Finally, aircraft landing gear generally allow for more than two degrees

of freedom; however only vertical motion is considered since the model is intended primarily for study of aircraft response to vertical runway excitation.

System Inputs

Two types of input are used for F(t). The first is the (1-cos wt) bump, where the frequency w is determined by dividing the velocity of the aircraft by the wave length λ of the bump and then multiplying by 2π. The argument of the cosine function is less than or equal to 2π., and for this input, the equations of motion are solved for the time duration of the bump. The second type of input is an equivalent deterministic input derived from runway spectra. It is a special function such that the runway roughness is represented as a random variable and root mean square system output values of force, displacement, velocity etc. can be analyzed. This input takes the form of a decaying exponential and is derived in Appendix B. For this kind of input, the equations of motion are solved over a long enough time period until no change in system response is detected by increasing the time period.

The decaying exponential derived from runway spectra is advantageous in that it does not require a range of frequencies to be covered as does the (1-cos wt) bump, and it provides a means to obtain root mean square system output values. However, runway spectral density is not a complete description of the random process. In a sense

it is obtained as an "averaging process" over very long lengths of terrain; therefore, a "discrete rut" or any kind of pothole in the runway is "averaged out". In addition, use of the decaying exponential input restricts analysis to a linear system as is verified in Appendix B; hence both the (1-cos wt) bump and the decaying exponential are used as system inputs to broaden the analysis. Illustrations of the (1-cos wt) bump and runway spectral density complying with (Mil-A-8862A) design criteria for runway roughness, are given in Figures 2 and 3.

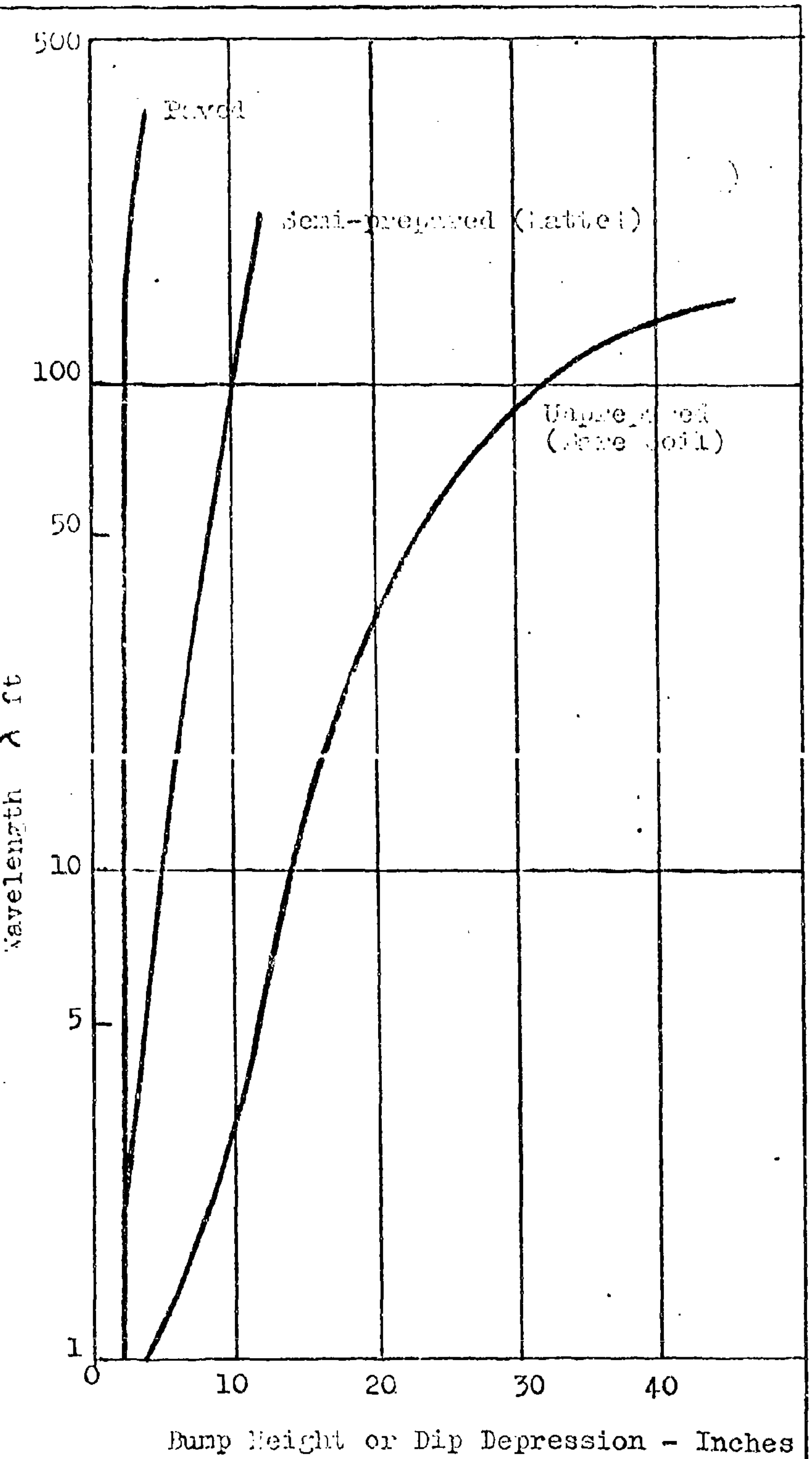


Figure 2

(1-cos wt) Bumps And Depressions

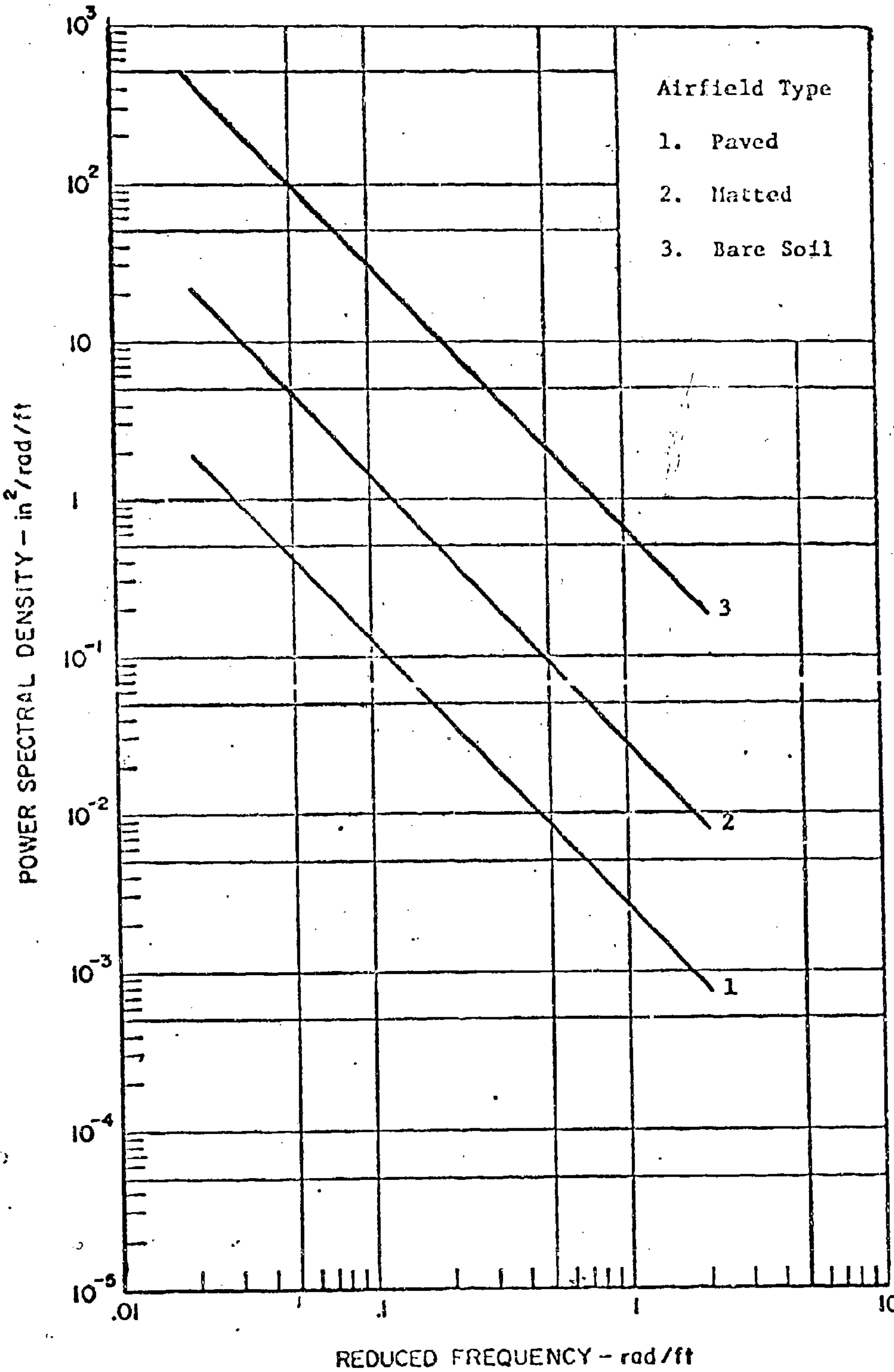

Airfield Type
1. Paved
2. Matted
3. Bare Soil
POWER SPECTRAL DENSITY – in²/rad/ft
REDUCED FREQUENCY – rad/ft
10³
10²
10
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
.01
.1
1
10
1
2
3

Figure 3

III. The Optimal Control Approach

Cost Criterion

An optimization criterion for an active shock and vibration system has usually been to minimize the absolute acceleration of the sprung mass, while holding the relative displacement or rattle space of the suspension system below some predetermined maxium value (Ref 1,2,3). However, large aircraft landing gear have as much as two to three feet allowable rattle space to absorb landing impact energy, while less than a half foot of displacement is required to absorb most runway imposed vibration energy, the displacement requirement being directly proportional to the bump or depression amplitude.

On the other hand, due to the relatively large mass of the airplane, there is the expected requirement of large control forces from the active element or actuator. It is therefore felt that as a first try, the optimization criterion should be to minimize the absolute acceleration of the sprung mass, the airplane, while at the same time placing a constraint on the control force instead of rattle space. In other words, the maximum control force obtainable is the constraint resulting from physical limitations of the hydraulic or pneumatic actuator used.

With the above criterion a system is visualized where the actuactor would function only in a rattle space

range of $\pm$ 6 in. allowing passive elements to set limitations on rattle space when the $\pm$ 6 in. range is exceeded. The active-passive combination system would seem especially attractive since it would retain the passive system in case of active system failure.

The optimization or minimization criteria can be expressed as:

$$\text{Minimize } J(u) = \int_0^{t_f} [\ddot{Z}^2 + PU(t)^2]\,dt \qquad (4)$$

where U(t) is the control function Fs(t)/F1 and $\ddot{Z}$ is the absolute acceleration defined by equation 3.

Actually, there are a number of mathematical formulations that could be used. If the maximum value of $\ddot{Z}$ is minimized along with a maximum value of rattle space X1, the solution would result in a bang-bang type of controller for U(t) (Ref 4,5). If the criterion is to minimize $\ddot{Z}$ in minimum time, the solution would again lead to a bang-bang type of controller for U(t). However, the bang-bang controller requires comparably larger control forces than the continuous duty controller, and it often demonstrates a poor ability to withstand broad band random inputs; thus, the bang-bang controller appears undesireable for landing gear use (Ref 3:14).

In contrast, the integral square criterion of equation (4) offers many practical advantages for the active landing gear system. The resulting system obtained from this criterion is linear and therefore easily analyzed, and a constant coefficient feedback system can be found and can usually be realized by an active system (Ref 3). The quadratic criterion is also advantageous in that a global minimum is assured. In other words, there is only one control function U(t) which satisfies the above criterion for a given system with a specified set of parameters. Finally, the integral square criteria is required when the equivalent deterministic input is used for the system must be linear before mean square output values can be theoretically predicted with ease from an input derived from runway spectral density (Ref Appendix B). Of course the criteria in equation 4 for use with the equivalent deterministic input can be stated:

$$\text{Minimize } J(u) = E(\ddot{Z}^2) + P\,E(u^2(t)) \qquad (5)$$

where the E ( ) notation denotes "expected value".

Theory

Equations 4 and 5 establish the cost criterion. To continue with the problem formulation, by substituting

equation 2 and 3 in equation 1, the system equations in state space notation are

$$\dot{X}_1 = X_2 \tag{6}$$

$$\dot{X}_2 = (1+R)g\,U(t) - Q_3X_1 - Q_4X_2 - Q_5(F(t) - X_3) \tag{7}$$

$$\dot{X}_3 = X_4 \tag{8}$$

$$\dot{X}_4 = -Rg\,U(t) + Q_6X_1 + Q_7X_2 + Q_5(F(t) - X_3) - g \tag{9}$$

where R, g, Q3, Q4, Q5, Q6, Q7, are defined in the list of symbols.

The Hamiltonian function (Ref 10) corresponding to the system differential equations 6-9 is

$$H = \sum_{i=0}^{n} P_i \dot{X}_i = P_0\left(\ddot{Z}^2 + P\,U^2(t)\right) + P_1X_2 \tag{10}$$

$$+ P_2 [(1+R) g u(t) - Q_3 X_1$$

$$- Q_4 X_2 - Q_5 (F(t) - X_3)]$$

$$+ P_3 X_4$$

$$+ P_4 [-R g u(t) + Q_6 X_1 + Q_7 X_2$$

$$+ Q_5 (F(t) - X_3) - g$$

where P_i is called the adjoint variable. It corresponds to the Lagrange multiplier in the conventional calculus of variations when equality constraints are present.

The system differential equations of the adjoint variable are derived as

$$\dot{P}_i = - \frac{\partial H}{\partial X_i} \quad i = 1,2 \tag{11}$$

and the transversality conditions are determined by

$$P_i(t_f) = \frac{\partial \Theta}{\partial X_i} [X_i(t_f), t_f] \quad i = 1,2 \tag{12}$$

where Θ is that part of the cost function which is a function only of final time t_f. In other words, in terms of the general cost formulation of the Bolza problem, the cost expressed in equation 4 contains no function Θ which is a function of t_f only; therefore, Θ is zero (Ref 10:57).

Finally, for a minimization problem, P_0 is a constant and greater than zero (Ref 10:64). The Hamiltonian is homogeneous in P_i, therefore P_0 can be set equal to one. For U(t) to be optimal, the Hamiltonian function H has to be a minimum with respect to the variable U(t). A necessary condition for optimality is then

$$\frac{dH}{dU} = 0 \tag{13}$$

The solution of equations 6-9 and 11-13 gives the optimal solution for U(t). A detailed mathematical formulation of the problem can be found in Appendix A.

IV. Numerical Techniques

Conjugate Gradient

The optimal control numerical technique used to solve the problem of this study is the conjugate gradient technique of Lasdon, Mitter and Warren (Ref 8). The technique is directly applicable only to unconstrained problems, but the constrained problem can be converted to an unconstrained problem through the use of a penalty function as is done in equations 4 and 5. The unconstrained problem under consideration is to minimize the absolute acceleration $\ddot{Z}$. At the same time, a penalty must be paid in acceleration $\ddot{Z}$ to constrain the control force U(t) below some predetermined value set by the physical limitations of the actuator.

The conjugate gradient method is a direct method. In other words, for an optimal solution, a search is made in the direction which directly minimizes the cost function. The gradient trajectory (Ref equation 13), its norm and the actual search direction are the only values which require computer storage. The search directions are formed from past and present values of the cost and its gradient. Then successive points are determined by linear minimization along the search directions which are always directions of descent. With search directions always descending, the conjugate gradient tends to converge

even from poor approximations to the minimum.

Algorithm

The conjugate gradient algorithm requires the computation of the gradient trajectory. Letting $U_o(t)$ be the first of approximations to the optimal control $U_i(t)$, $i = 1,2$ etc., then the corresponding gradient $G_o(u)$ is computed by solving the state equations 6-9 forward in time, the adjoint equations 11 backward in time, and obtaining the gradient dH/dU from equation 13.

The algorithm proceeds as follows:

1. Guess $U_o(t)$, an arbitrary selection of $U_o(t) = o$ was made
2. Integrate state equations forward and adjoint equations backwards
3. Compute gradient $G_o = G_o(U_o)$ from equation 13
4. Compute search direction $S_o = -G_o$
5. Choose $\alpha = \alpha_i$ to minimize $J(U_i + \alpha_i S_i)$
6. $U_{i+1} = U_i + \alpha_i S_i$
7. $G_{i+1} = G(U_{i+1})$
8. $B_i = (G_{i+1}, G_{i+1})/(G_i, G_i)$
9. $S_{i+1} = -G_{i+1} + B_i S_i$

   where $(G_i, G_i) = \int_0^{t_f} G_i^2(t)\, dt$

10. Set $i = i + 1$, go to 5

Steps 5-10 are repeated integrating the state equations forward and adjoint equations backward for each computation of the gradient, until the change in cost is negligible.

Alpha Search

The alpha value indicated in step 5 of the algorithm is determined by initially assigning a small value - about .1 - to alpha and then checking the inner product $G_{i+1} S_i$. If the sign of the inner product is positive with the initial alpha value, a tenth of the initial value is repeatedly taken until a value of alpha is found which gives a negative inner product. Then the alpha value is repeatedly doubled until the inner product is positive. Cubic interpolation is then used to determine the value of alpha which gives a zero inner product. The inner product or directional derivative is the slope of the cost with respect to alpha; therefore, the alpha value with a corresponding zero inner product is the alpha value which gives the lowest cost in that search direction.

Some alpha searches use one over the norm of the search direction S as a guide for the first initial alpha guess (Ref 9). However after trying this, it was found from running a number of sample problems (Ref 8:136) and the dynamics of the thesis problem that a guess in the range of 0.001 to 0.1 nearly always gave a negative

inner product and was generally better than that of one over the norm of the search direction. A better alpha guess, of course means a savings in computer iteration time. The computer program can be found in Appendix E.

V. Results

All results are based on the following arbitrarily selected parameters for the dynamic model.

Cs	100 lb sec/ft
F1	100,000 lb
F2	1,000 lb
Ks	200,000 lb/ft
Kt	1200,000 lb/ft

The response of the dynamic model to the (1-cos wt) bump input is illustrated in Figures 4-17. The plots show the control force U(t) required and the acceleration $\ddot{Z}$ produced when the weighting factor P is set equal to 0, 1, 10 and infinity. The value of P equal to zero illustrates the virtually zero acceleration of the aircraft produced when there is no limit placed on the control force U(t). With P equal to one, equal weighting is then given in the cost function to minimize both the control force U(t) and the acceleration $\ddot{Z}$ (see Appendix A). With a P value of 10, control is weighted even more until finally with P equal to infinity, the control force is then zero and the resultant acceleration is the acceleration produced by a completely passive system.

Figures 4-17 cover the frequency range of .5 to 32 Hz geometrically by doubling the frequency of the argument of the cosine function beginning at .5 Hz. The bump height or amplitude of the (1-cos wt) function is 2.5 in.

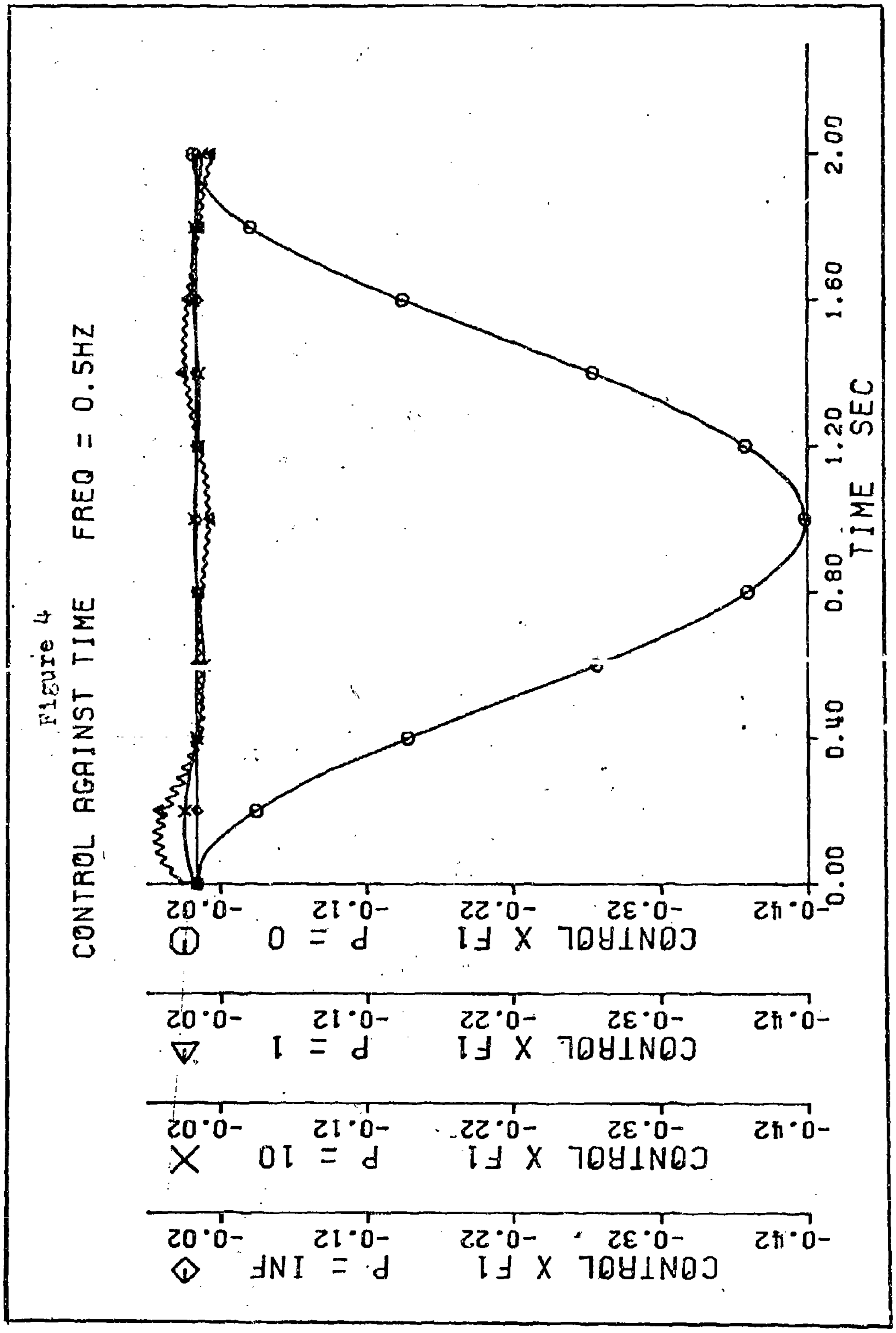
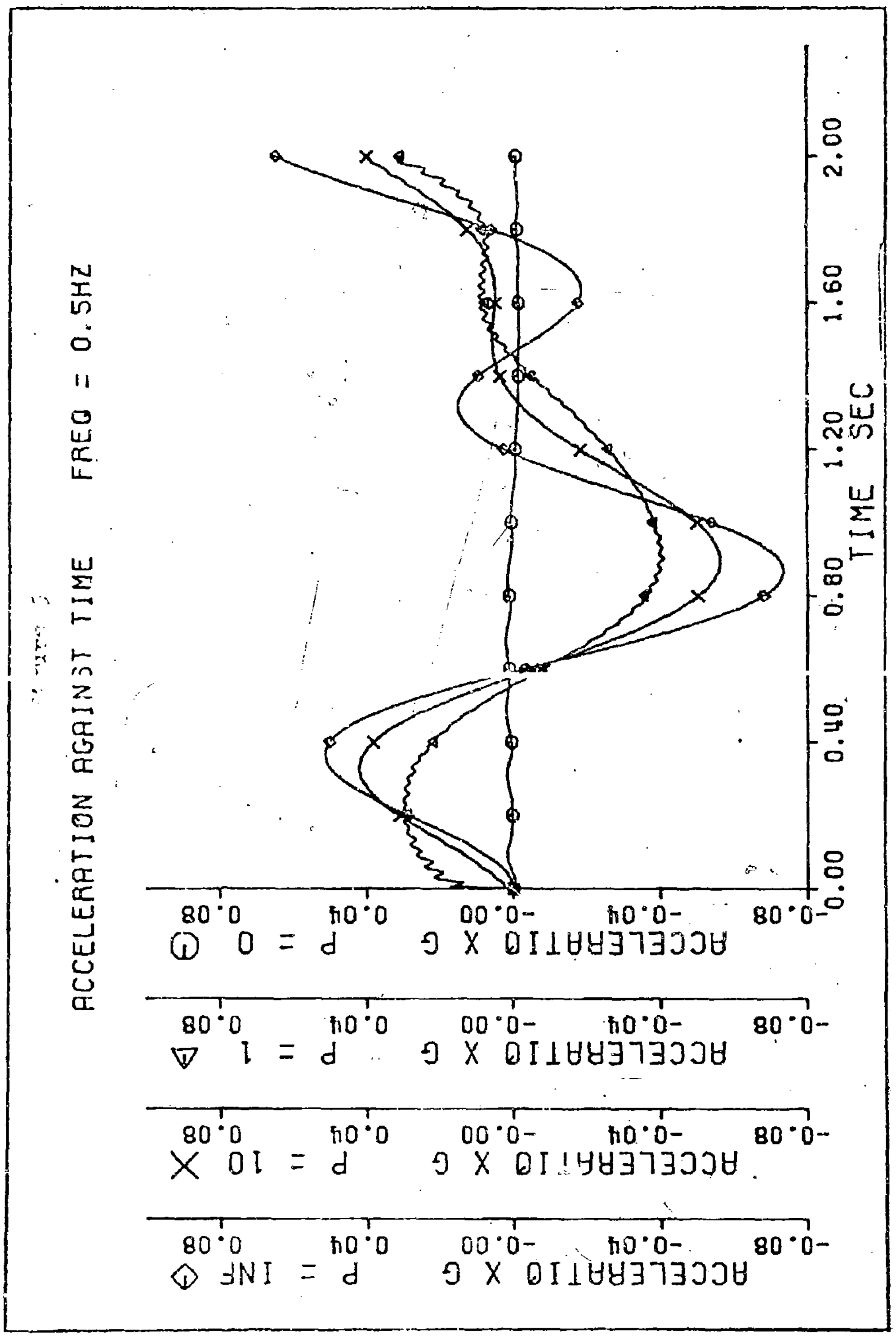

CONTROL AGAINST TIME FREQ = 0.5HZ
TIME SEC
0.00
0.40
0.80
1.20
1.60
2.00
CONTROL X F1 P = 0
CONTROL X F1 P = 1
CONTROL X F1 P = 10
CONTROL X F1 P = INF
-0.02
-0.12
-0.22
-0.32
-0.42

Figure 4


ACCELERATION AGAINST TIME   FREQ = 0.5HZ
TIME SEC
0.00
0.40
0.80
1.20
1.60
2.00
ACCELERATIO X G   P = 0
ACCELERATIO X G   P = 1
ACCELERATIO X G   P = 10
ACCELERATIO X G   P = INF
-0.08
-0.04
-0.00
0.04
0.08

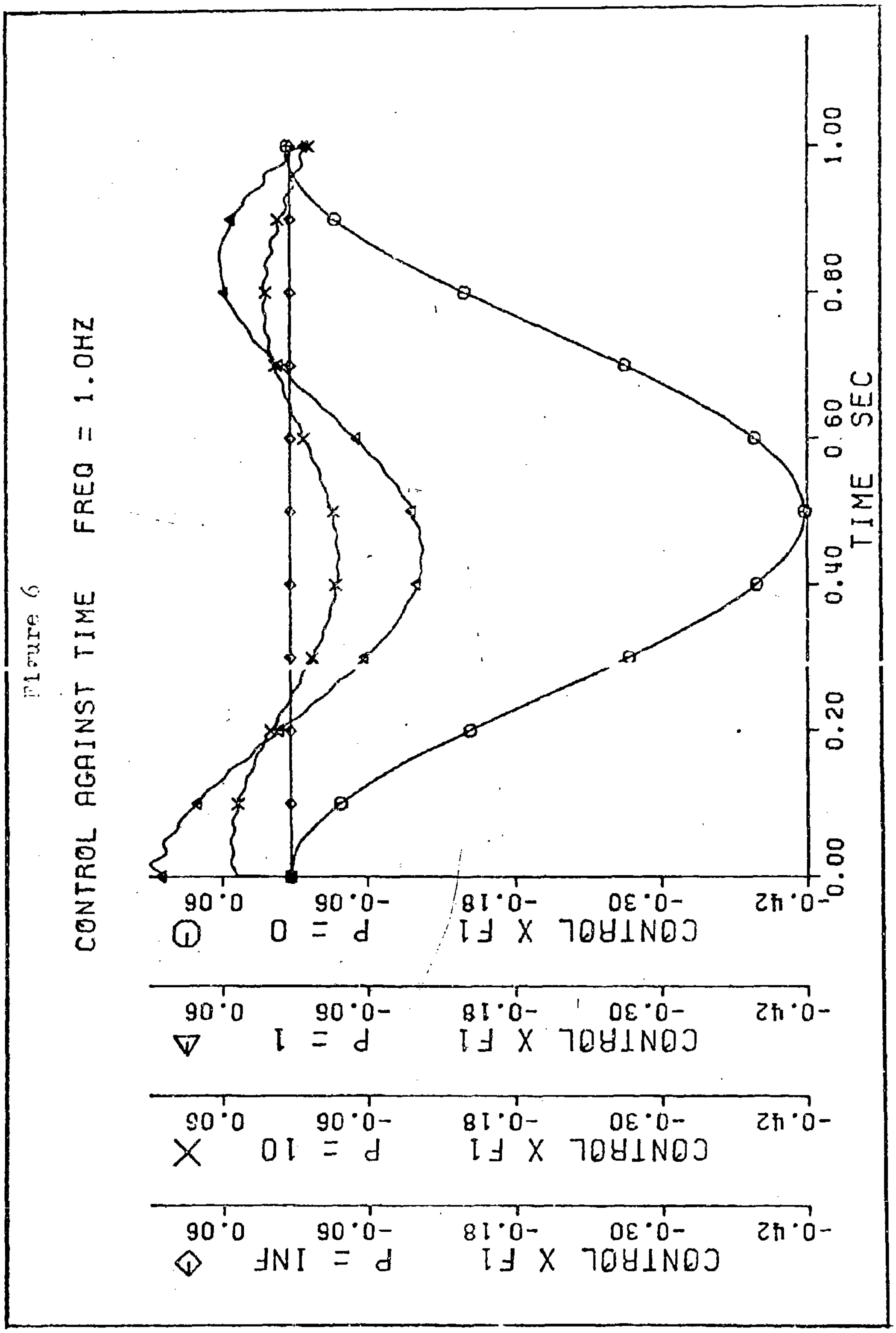

CONTROL AGAINST TIME FREQ = 1.0HZ
TIME SEC
0.00
0.20
0.40
0.60
0.80
1.00
CONTROL X F1 P = 0
CONTROL X F1 P = 1
CONTROL X F1 P = 10
CONTROL X F1 P = INF
-0.42
-0.30
-0.18
-0.06
0.06

Figure 6

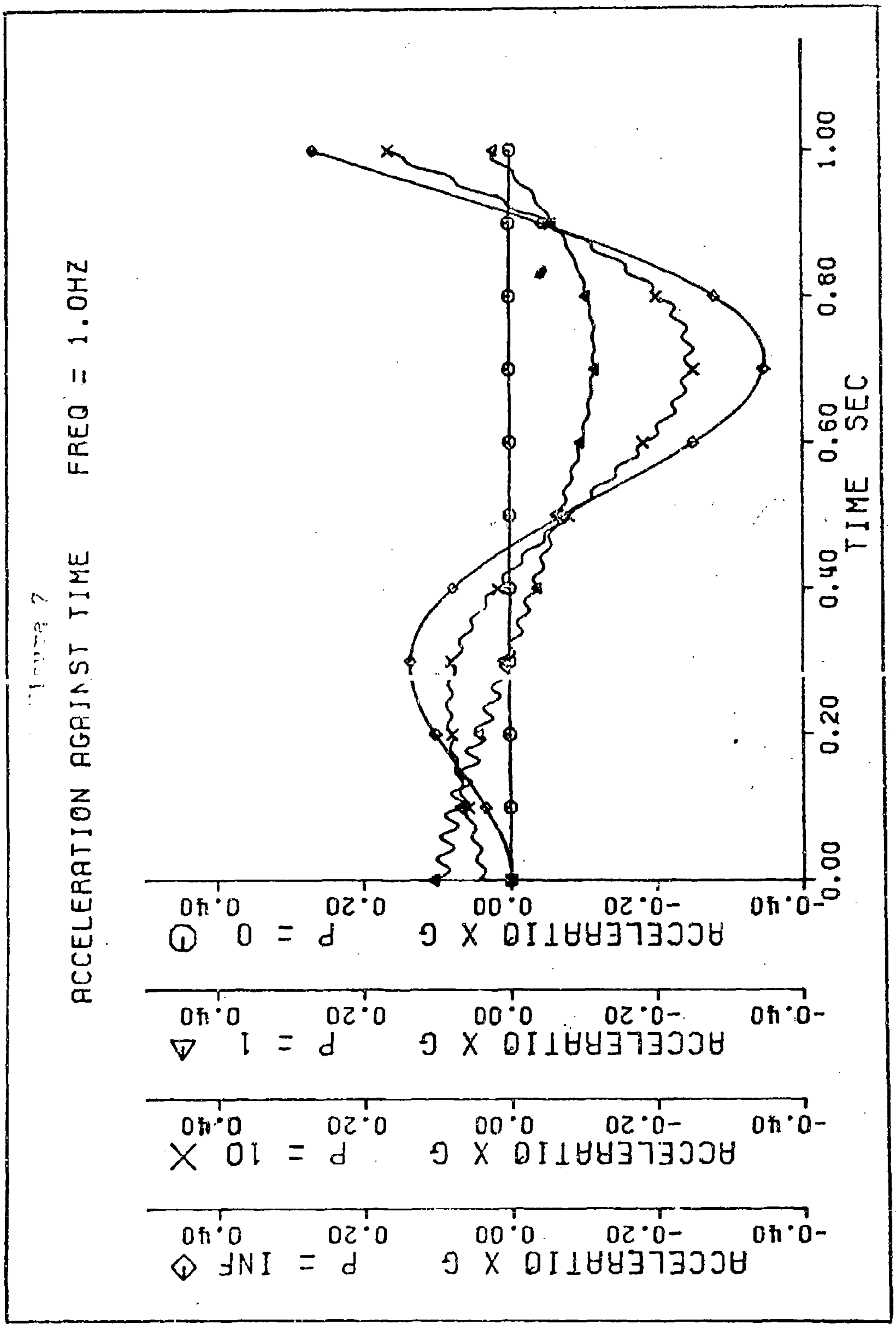

ACCELERATION AGAINST TIME FREQ = 1.0HZ
ACCELERATIO X G P = 0
ACCELERATIO X G P = 1
ACCELERATIO X G P = 10
ACCELERATIO X G P = INF
-0.40
-0.20
0.00
0.20
0.40
0.00
0.20
0.40
0.60
0.80
1.00
TIME SEC

Figure 7

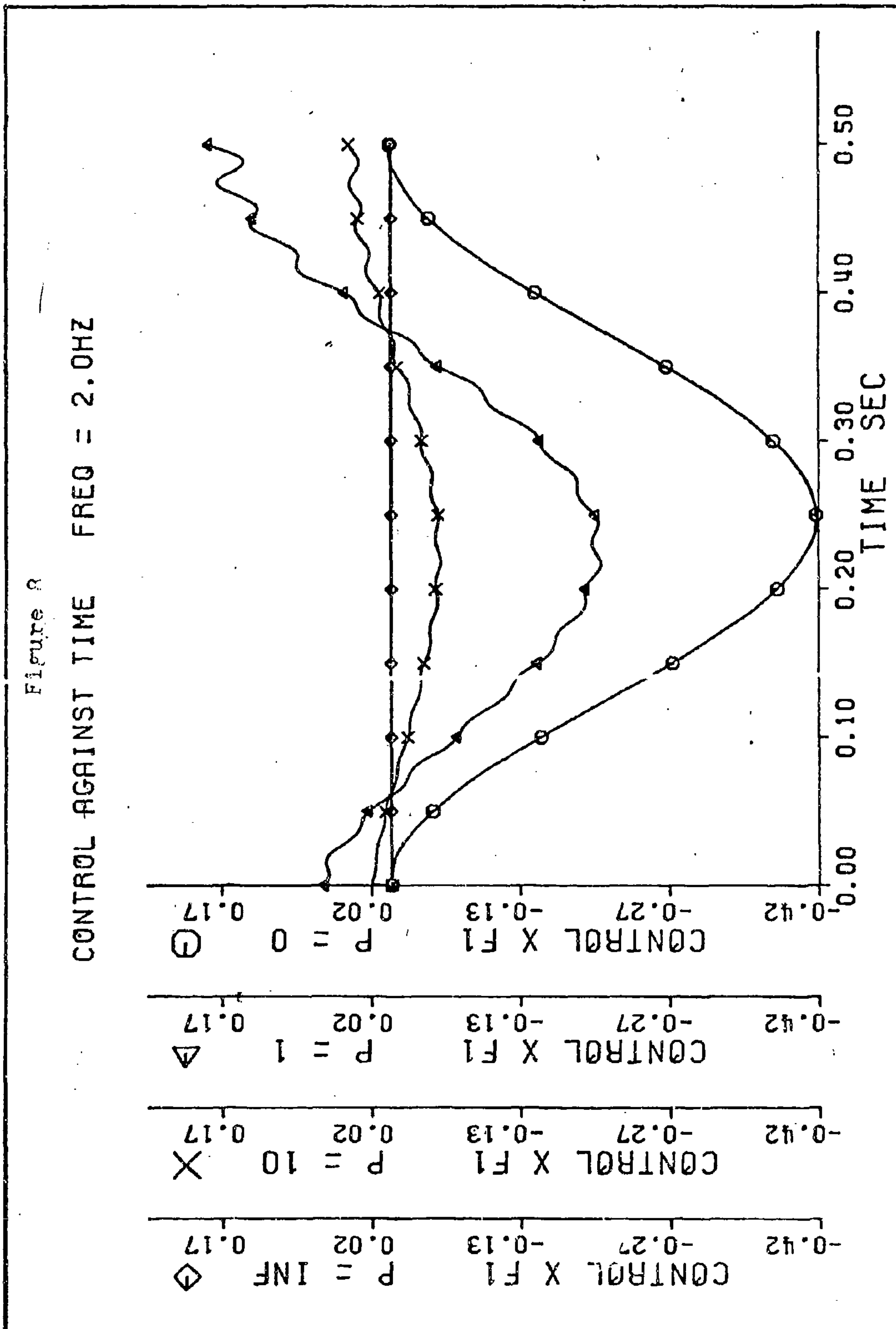

CONTROL AGAINST TIME   FREQ = 2.0HZ
TIME SEC
0.00
0.10
0.20
0.30
0.40
0.50
CONTROL X F1   P = 0
CONTROL X F1   P = 1
CONTROL X F1   P = 10
CONTROL X F1   P = INF
-0.42
-0.27
-0.13
0.02
0.17

Figure 8

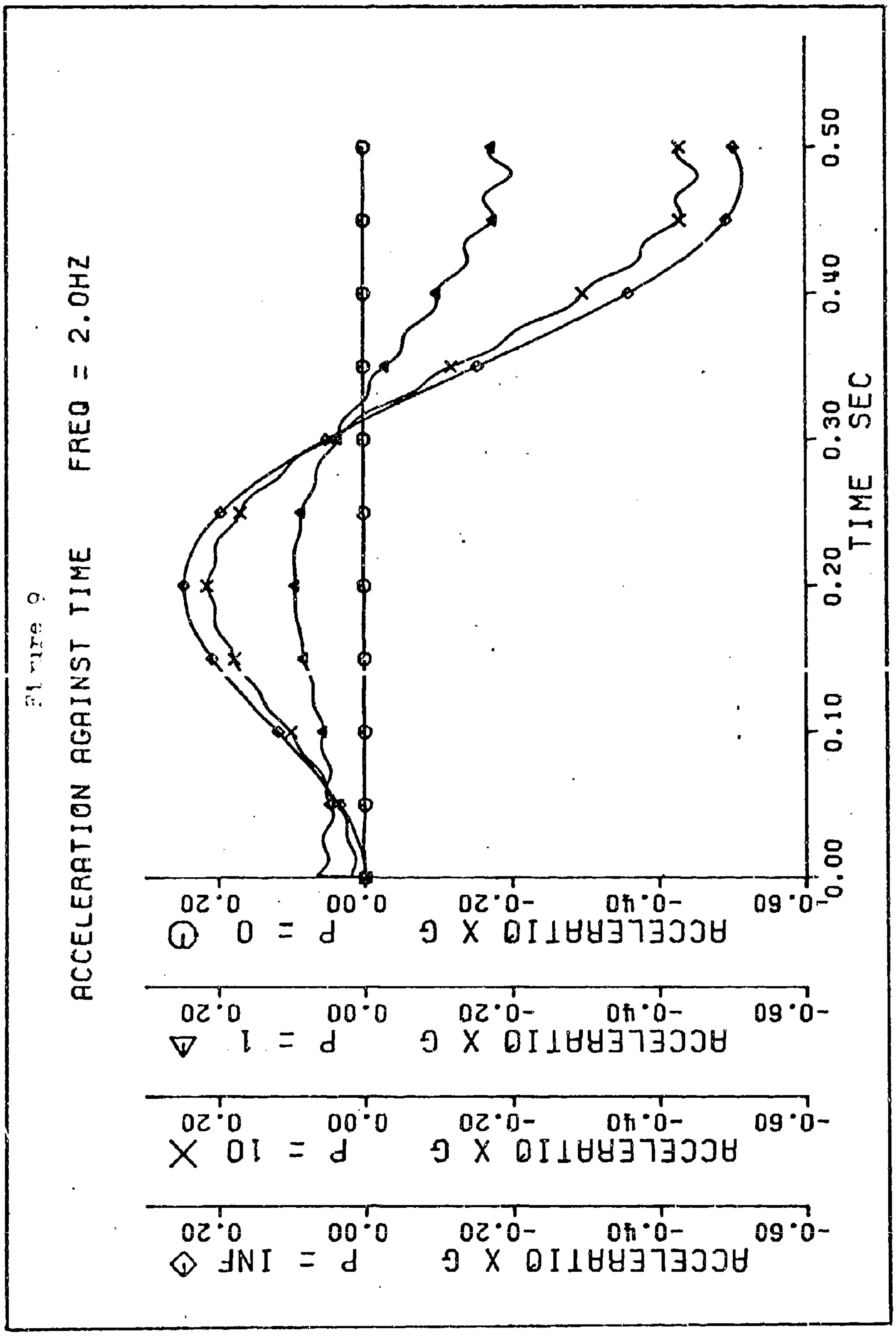


Figure 9
ACCELERATION AGAINST TIME FREQ = 2.0HZ
TIME SEC
0.00 0.10 0.20 0.30 0.40 0.50
ACCELERATIO X G P = 0
ACCELERATIO X G P = 1
ACCELERATIO X G P = 10
ACCELERATIO X G P = INF
-0.60 -0.40 -0.20 0.00 0.20

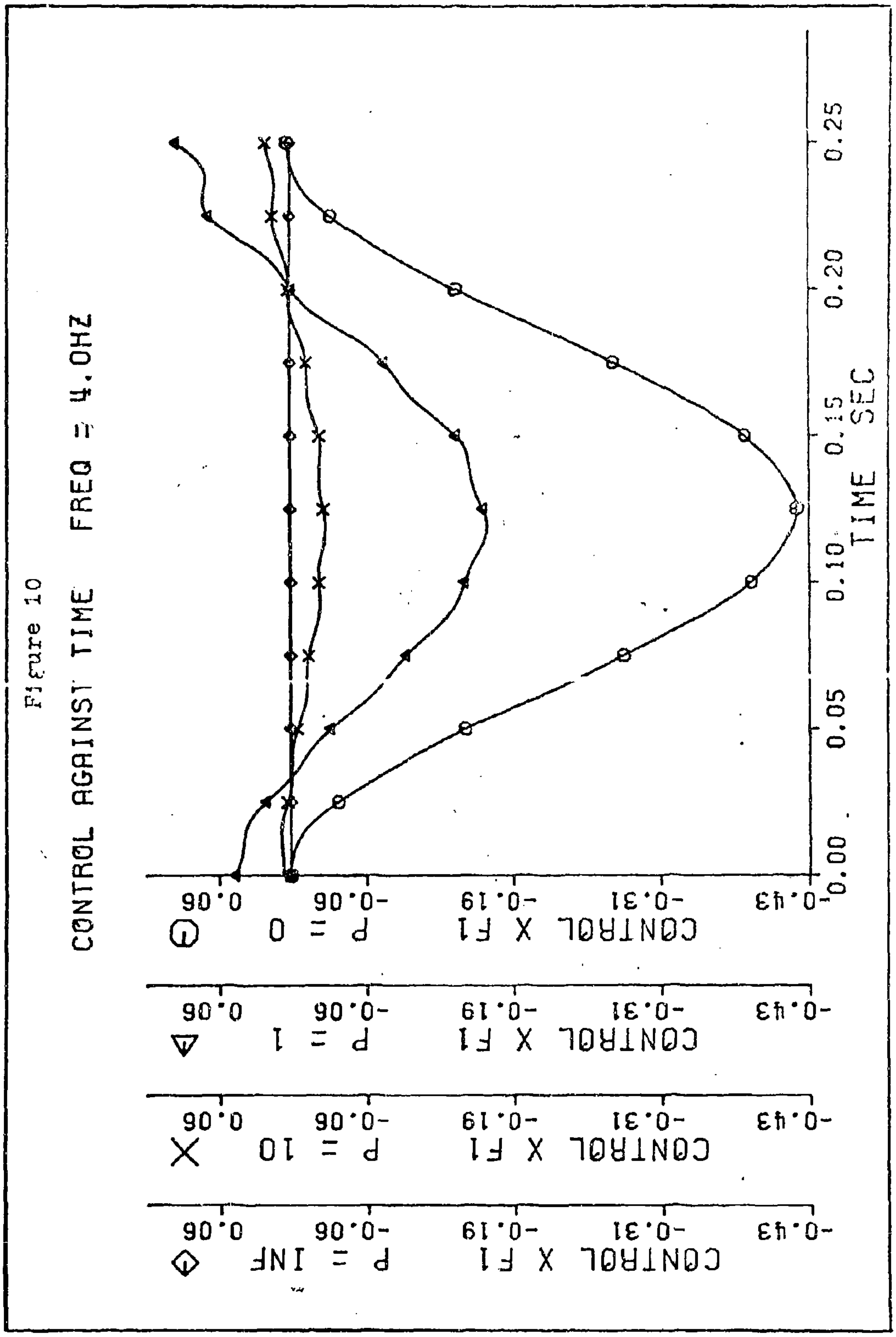

CONTROL AGAINST TIME   FREQ = 4.0HZ
0.00
0.05
0.10
0.15
0.20
0.25
TIME SEC
-0.43
-0.31
-0.19
-0.06
0.06
CONTROL X F1   P = 0
CONTROL X F1   P = 1
CONTROL X F1   P = 10
CONTROL X F1   P = INF

Figure 10

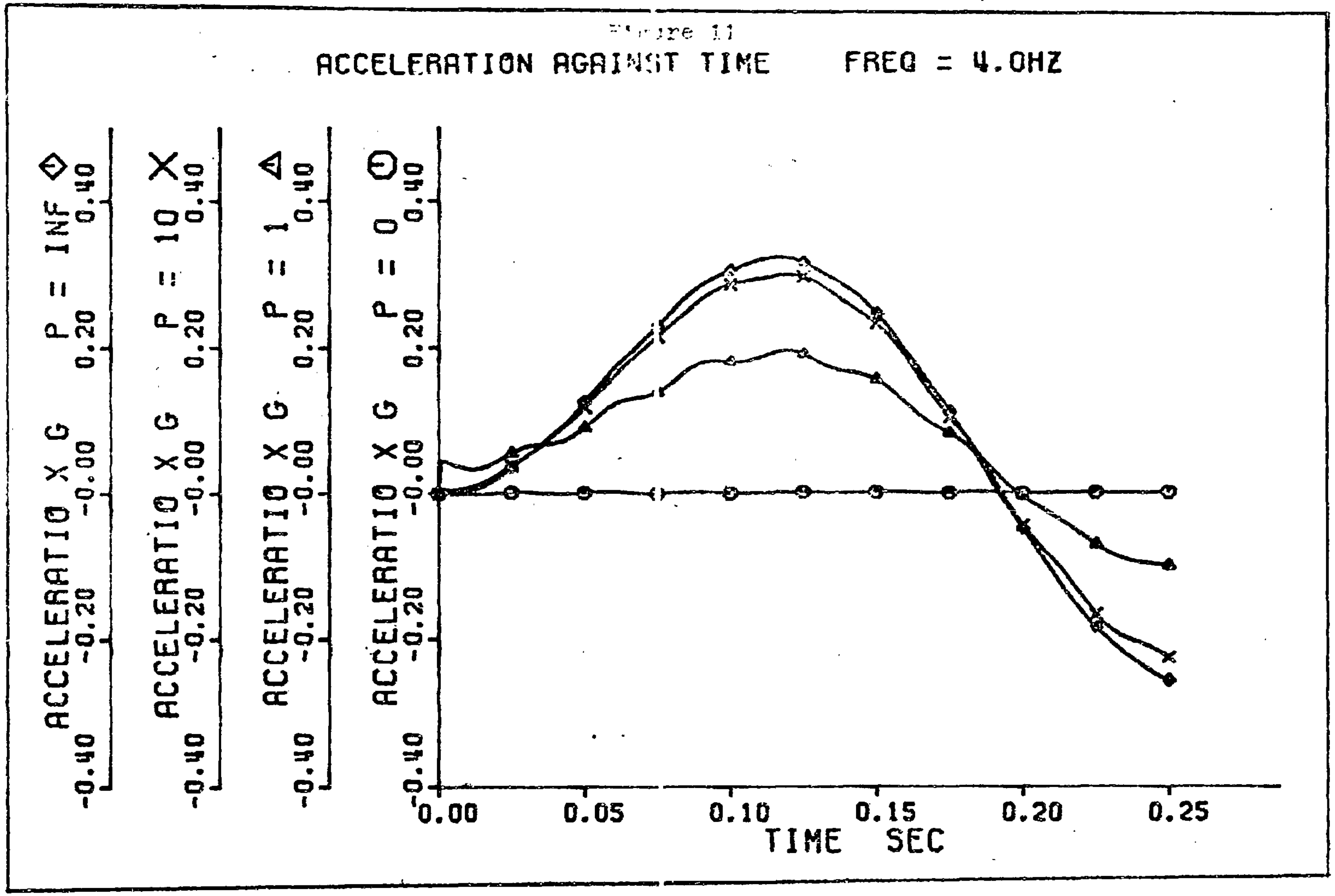


Figure 11
ACCELERATION AGAINST TIME FREQ = 4.0HZ
ACCELERATIO X G P = INF
ACCELERATIO X G P = 10
ACCELERATIO X G P = 1
ACCELERATIO X G P = 0
-0.40
-0.20
-0.00
0.20
0.40
0.00
0.05
0.10
0.15
0.20
0.25
TIME SEC

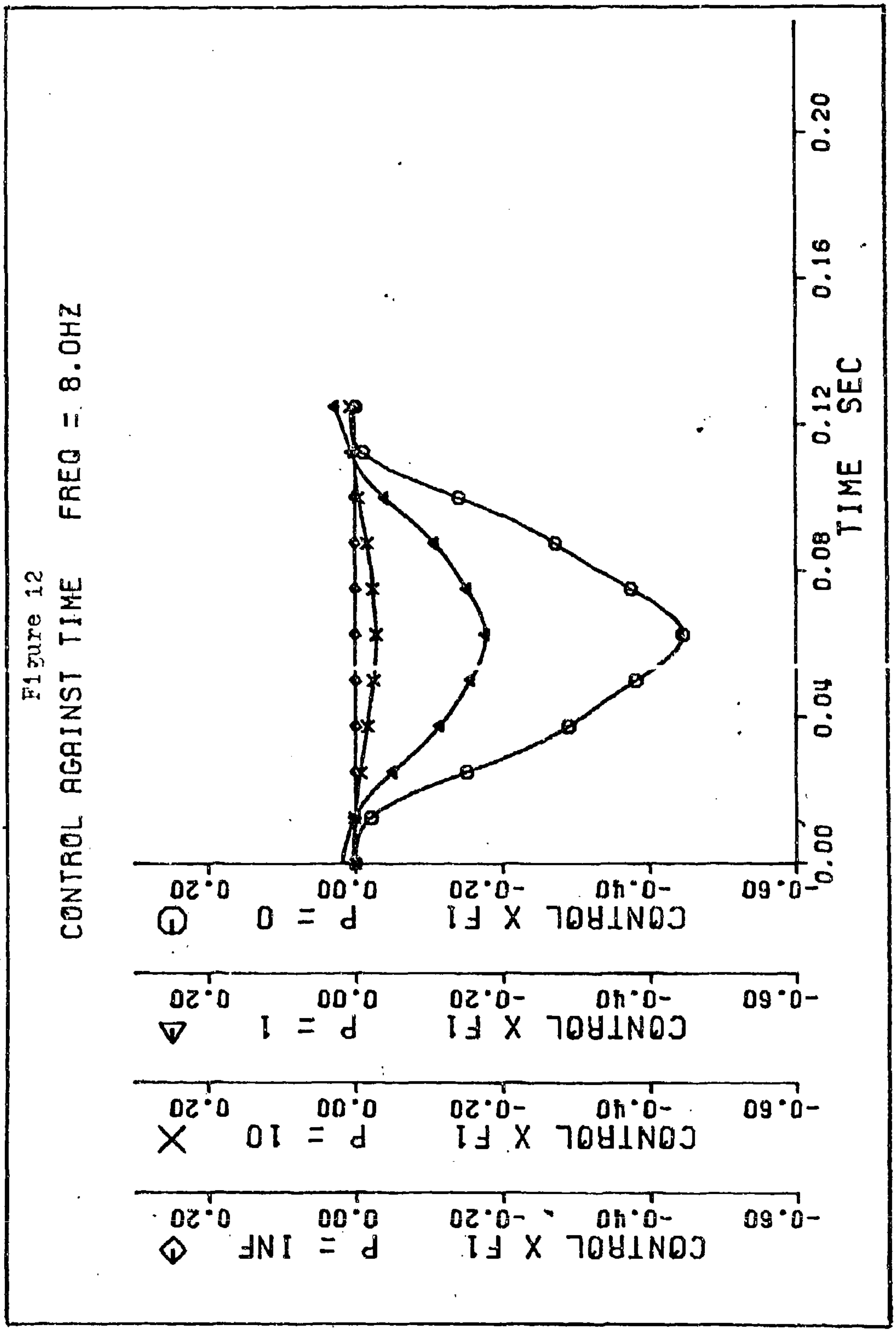
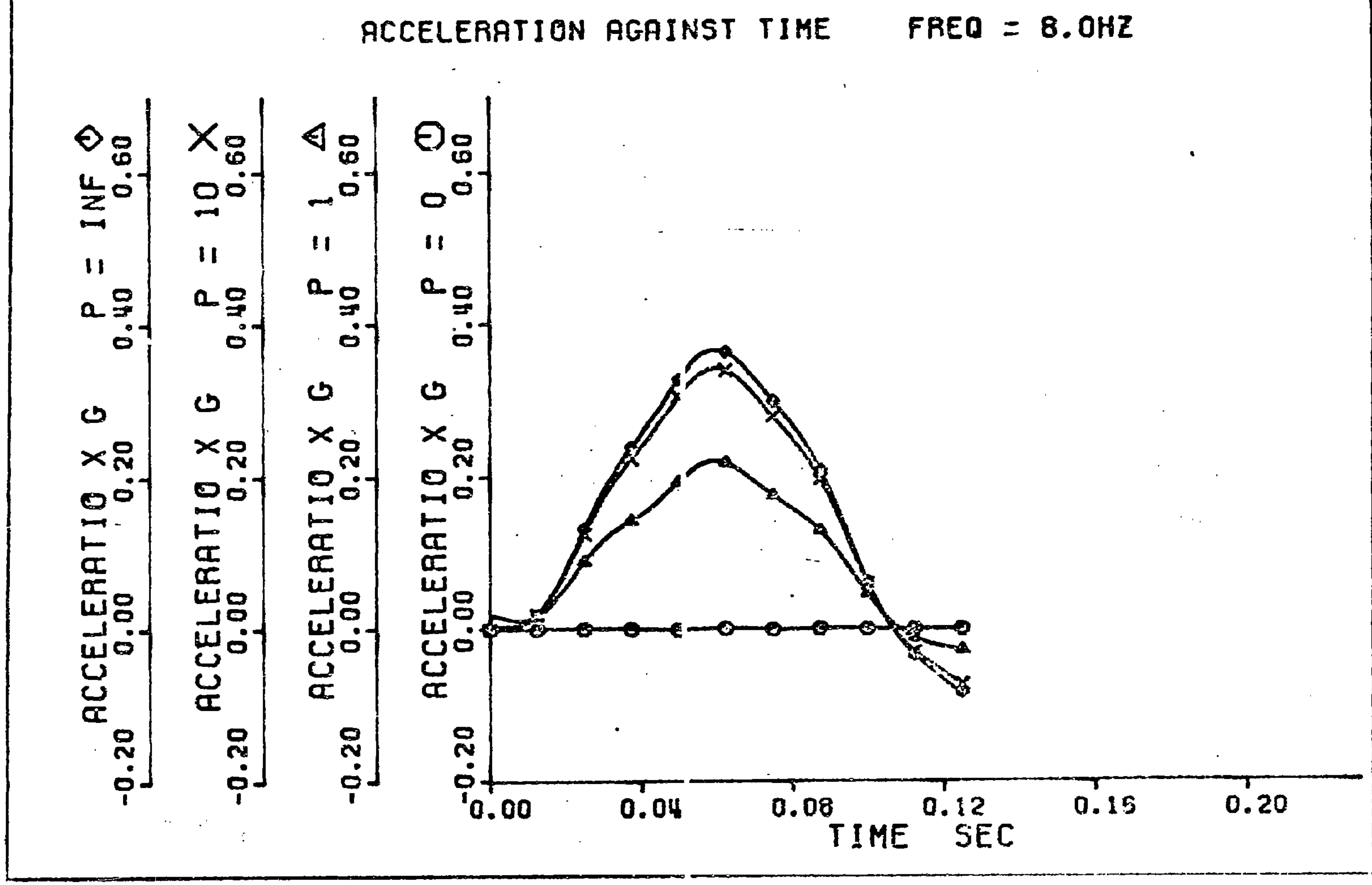


Figure 12

CONTROL AGAINST TIME FREQ = 8.0HZ

Figure 13


ACCELERATION AGAINST TIME FREQ = 8.0HZ
ACCELERATIO X G P = INF
ACCELERATIO X G P = 10
ACCELERATIO X G P = 1
ACCELERATIO X G P = 0
-0.20
0.00
0.20
0.40
0.60
0.00
0.04
0.08
0.12
0.16
0.20
TIME SEC

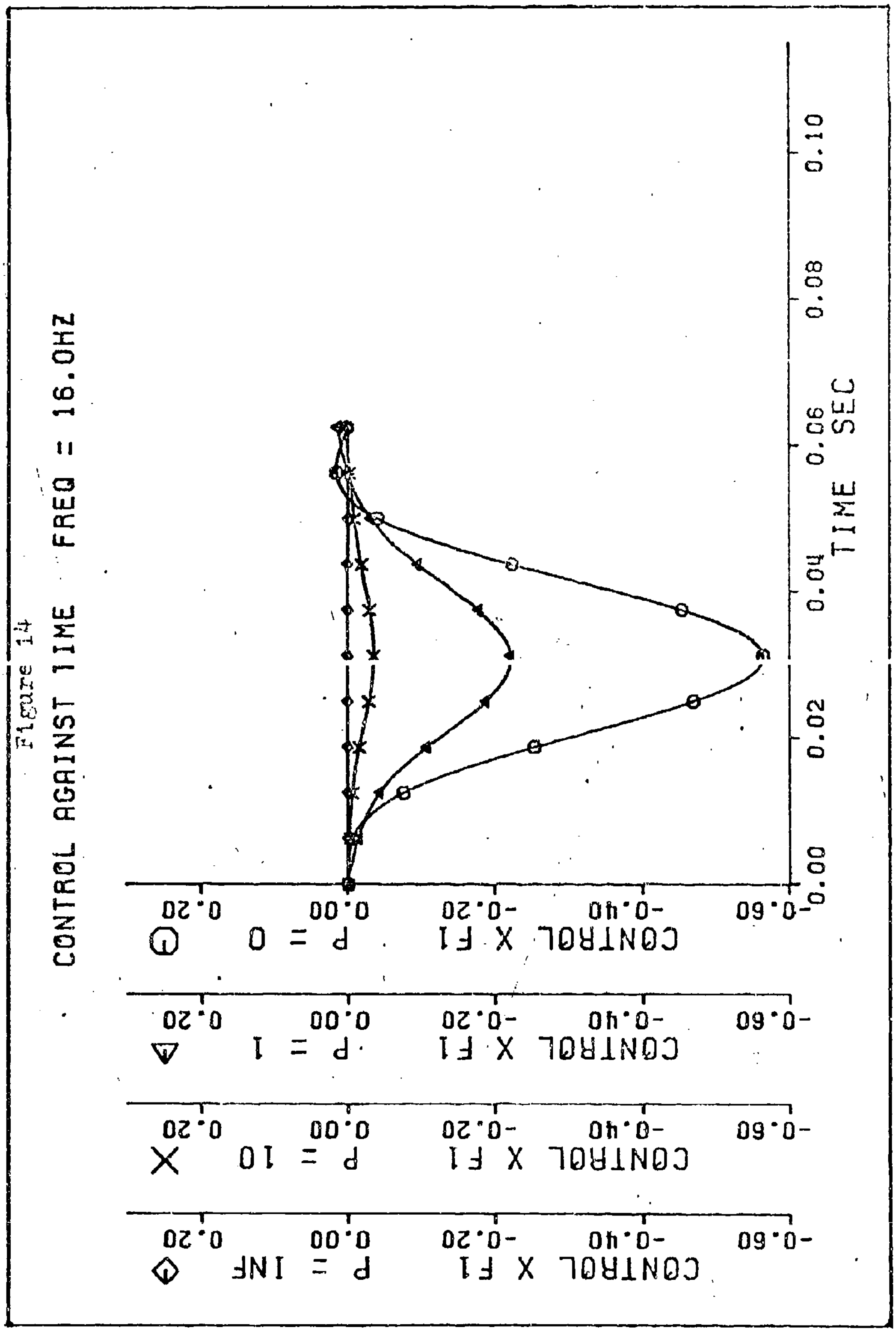

CONTROL AGAINST TIME FREQ = 16.0HZ
TIME SEC
0.00
0.02
0.04
0.06
0.08
0.10
CONTROL X F1 P = 0
CONTROL X F1 P = 1
CONTROL X F1 P = 10
CONTROL X F1 P = INF
-0.60
-0.40
-0.20
0.00
0.20

Figure 14

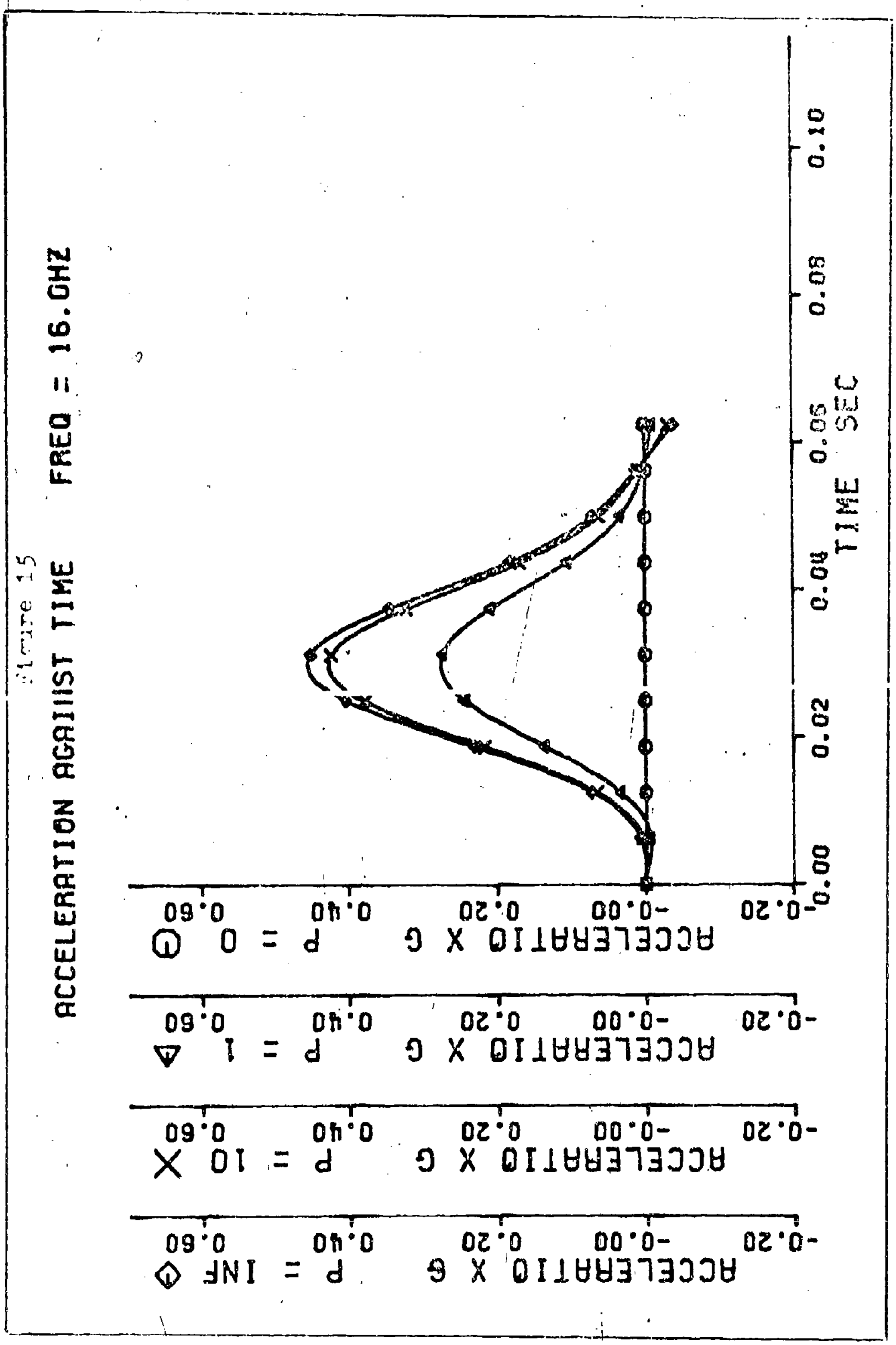


Figure 15
ACCELERATION AGAINST TIME
FREQ = 16.0HZ
TIME SEC
0.00
0.02
0.04
0.06
0.08
0.10
ACCELERATIO X G
P = 0
P = 1
P = 10
P = INF
-0.20
-0.00
0.20
0.40
0.60

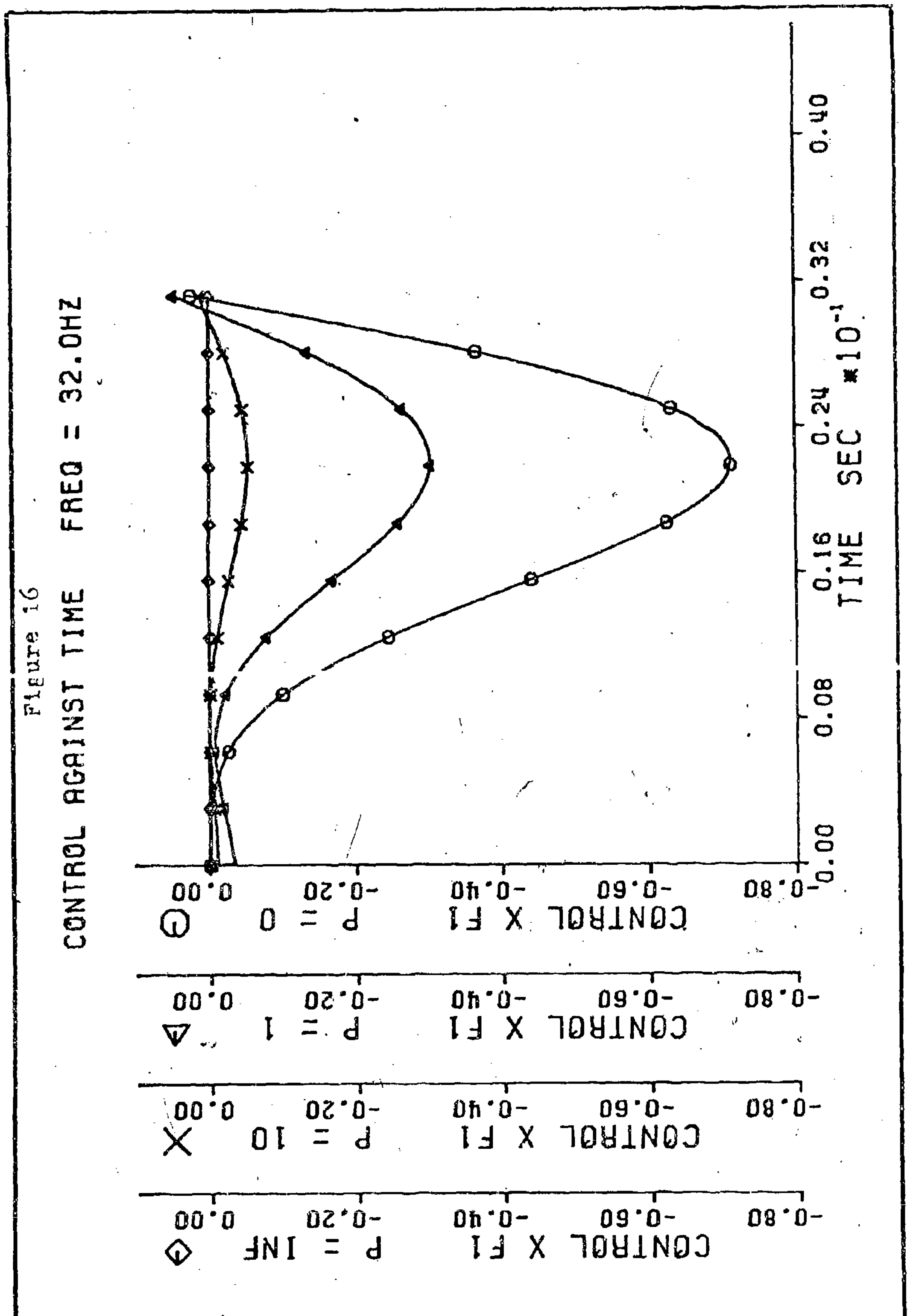
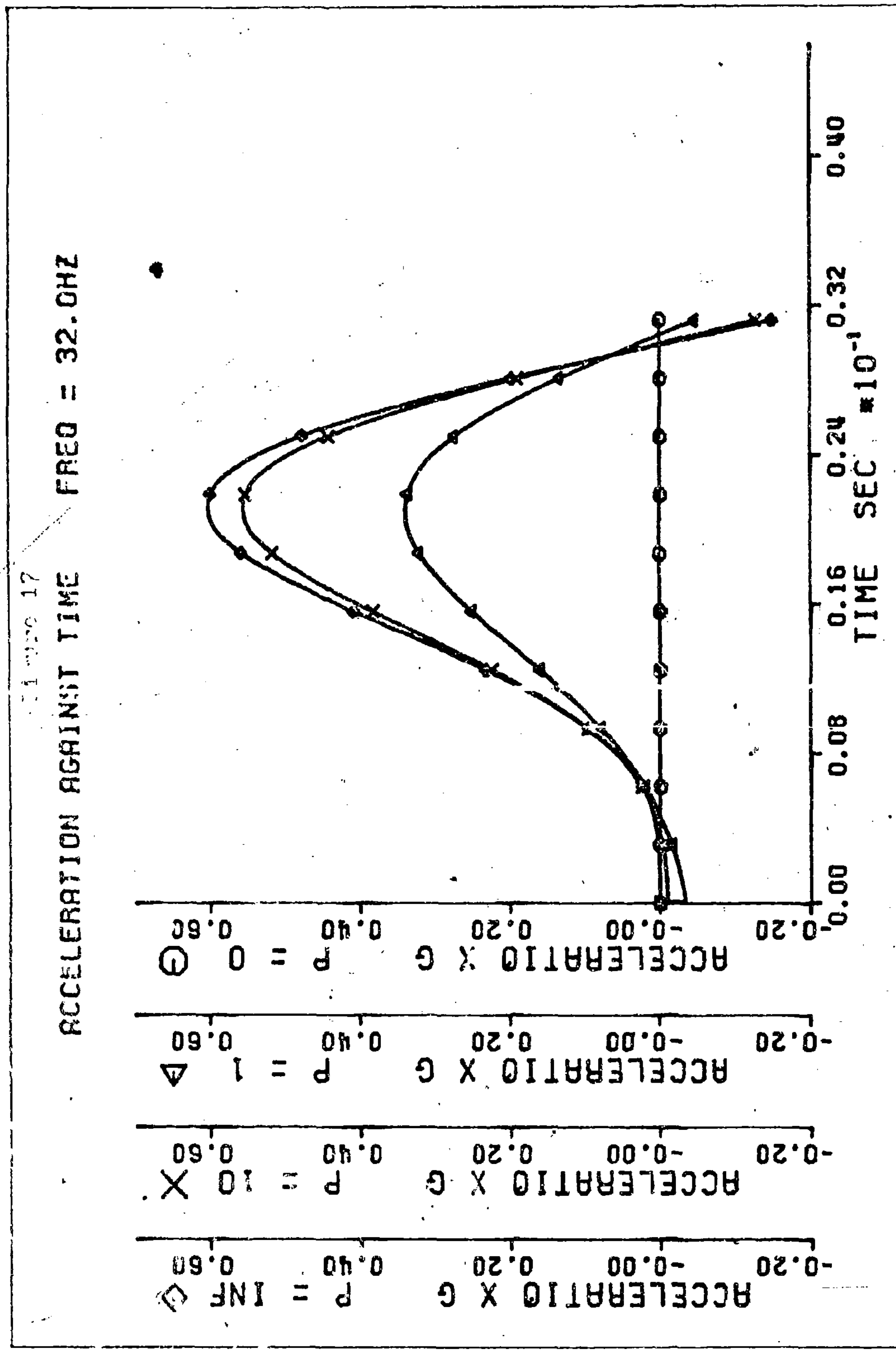

CONTROL AGAINST TIME FREQ = 32.0HZ
TIME SEC *10^-1
0.00
0.08
0.16
0.24
0.32
0.40
CONTROL X F1 P = 0
CONTROL X F1 P = 1
CONTROL X F1 P = 10
CONTROL X F1 P = INF
0.00
-0.20
-0.40
-0.60
-0.80

Figure 16


ACCELERATION AGAINST TIME
FREQ = 32.0HZ
ACCELERATIO X G P = 0
ACCELERATIO X G P = 1
ACCELERATIO X G P = 10
ACCELERATIO X G P = INF
-0.20
-0.00
0.20
0.40
0.60
0.00
0.08
0.16
0.24
0.32
0.40
TIME SEC *10^-1

corresponding to the paved runways in Figure 2. The figures are arranged in pairs such that the control force required for the four values of P for a particular frequency bump is given, and then on the following page, the corresponding acceleration is given for the same values of P and same bump frequency.

One observation that can be made is the increase in control force required as the bump frequency increases. This is reasonable, since the faster the aircraft moves over the bump, the faster the actuator must contract the landing gear and then extend the landing gear to maintain the F1 force at all times. The higher aircraft velocity also accounts for the increase in force required with increase in bump frequency around the final portion of the bump because again as the aircraft moves over the bump, the actuator must extend the landing gear faster to maintain the aircraft at a fixed level.

A second observation is the waviness in the plots at the lower frequencies with the weighting factor P equal to one. The weighting of P equal to one tends to make the cost function work against itself because it is being asked to find the U(t) to minimize $\ddot{Z}$ but at the same time keep U(t) small. These two values of U(t) of course ar[illegible] opposite ends of the scales. In addition and even more important, the dominant resonant frequency of the system is located at 1.2 Hz which does not permit the

computer program to converge to as low a cost near 1 Hz as reached at other bump frequencies.

A third observation of course is the marked improvement of the active system over the passive system. With P equal to infinity, there is considerable vertical acceleration to the aircraft, but with P equal to zero, the acceleration is virtually zero meaning that as long as the actuator can produce the force required there will be little or no acceleration to the aircraft.

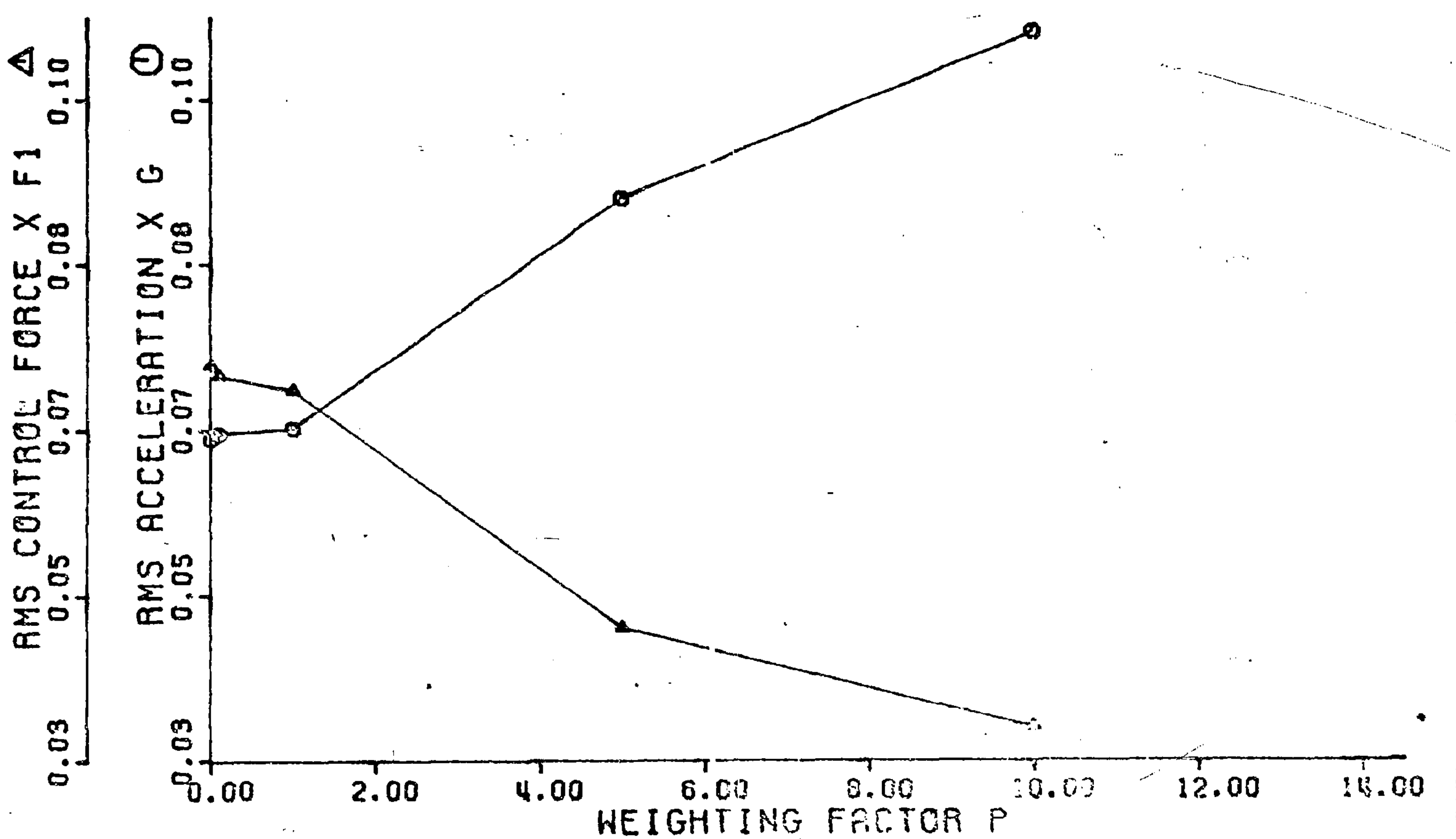
For the decaying exponential input, a plot of the root mean square (RMS) values of airplane acceleration and actuator control force against the weighting factor P is given in Figure 18. The RMS values are obtained by taking the square root of the integral square value of acceleration $\ddot{Z}$ and the square root of the integral square value of control force U(t). Mathematically it means evaluating equation 4 as two separate integrals without the weighting factor P.

The RMS values for the acceleration and control for P equal to infinity are not shown on the plot as in the (1-cos wt) plots because the value of acceleration for the totally passive system would be off the paper. The RMS value of acceleration at P equal to infinity is .543g and the RMS control for this value of P is .000169 x F1 or virtually zero.

For the RMS plot, the decaying exponetial input is

Figure 18

RMS ACCELERATION AND CONTROL AGAINST P



derived from the runway spectral density of the form $AV/(s^2+(2\pi V/\lambda_o)^2)$. The A is a parameter equal to 10^{-5} for most runways (Ref 1:49), and V is the velocity of the aircraft which is set equal to 200 ft/sec to represent a maximum take off velocity. The $2\pi V/\lambda_o$ term represents the break frequency at which the runway spectra level off. The runway spectra, of course must level off due to the finite height of runways. Using an RMS value of runway height of .5 ft (Ref 1:14), and the above spectral density form, the value of the wave length λ_o is found to be 5×10^4 ft from equation 33 in Appendix B. The above values of A, V, and λ_o are then substituted in equation 36 of Appendix B to obtain the exponential input $\sqrt{2\pi AV}\ EXP(-2\pi Vt/\lambda_o)$ used for the Figure 18 plot.

The plot reveals the trade-off between RMS acceleration $\ddot{Z}$ and control force U(t). The maximum value of RMS control force required is of course where P equals zero. This value is approximately 7200 lb. In other words the 7200 lb is the standard deviation of U(t) or σ_u . Therefore given a normal distribution, $3\sigma_u$ or a 21600 lb force would be the maximum force required from the actuator to ensure that the probability of actuator saturation would be less than .3 percent. Like the (1-cos wt) input plots, the RMS plot also reveals the better vibration isolation performance of the active system over the

passive system. As the weighting factor P increases, the acceleration becomes as high as .543g with the completely passive system at P equal to infinity, but with the active controller in the system the acceleration can be kept below an RMS value of .07g.

Numerical Aspects

For the (1-cos wt) input the computer program converged in 4 to 5 iterations in approximately 3 minutes time. One exception was the convergence rate around the system resonant frequency of 1.2 Hz. Near this frequency, a U(t) guess of zero or other constant value guess would require 15 to 20 minutes computer time to converge to approximately the same cost values obtained at frequencies located away from the resonant frequency. However, convergence near the resonant frequency was as fast as that of other frequencies if the solution for U(t) found away from the resonant frequency was used for frequencies near the resonant frequency.

With this procedure, the number of iterations and the time to converge were reduced to 5 iterations and 3 minutes computer time respectively, but the cost values obtained near the resonant frequency were still not quite as low as those of other frequencies, as is evident by the waviness of the 1,2, and 4 Hz plots of the (1-cos wt) bump input. At these frequency inputs the accuracy of the values of U(t) and $\ddot{Z}$ fell from 3 significant figures to

about one significant figure or a "ball park" value. Possibly in the resonant frequency area a "second variational" numerical technique would give better solutions for U(t) and $\ddot{Z}$ using the gradient solution for U(t) as an estimate (Ref 10).

Another significant factor which effected the convergence rate was the definition of U(t). At first U(t) was defined as F(t)/W1. This resulted in weightings in the dynamics of the model with ratios of 1 to 12g, that is, the weighting of U(t) would be 1 while that of another term might be 12g. In contrast, by defining U(t) to be F(t)/F1 (see Appendix A) the highest weighting ratio between any two terms became 1g to 12g. The end result of this definition was that the number of iterations and the time for convergence was cut in half. For example, for the (1-cos wt) inputs the average number of iterations for convergence was reduced from 10 to 5.

The convergence time for the decaying exponential input was considerably longer than that of the (1-cos wt) input. It generally fell in the range of 20 to 30 minutes for various values of the weighting factor P. The longer convergence time was expected since the decaying exponential input had a very slow decay rate which required the model dynamics to be solved over a comparably greater real time than that required by the (1-cos wt) input. However, it was still felt that maybe the 4th order Runge-Kutta integration procedure in Appendix C was not

sufficiently accurate, there-by causing a longer convergence time. Consequently, another gradient program was written using a prewritten integration routine and a function minimization routine already on hand. The function minimization routine was also modeled after the conjugate gradient method as the one in Appendix C.

The second program required more time per iteration, while the cost decrease was greater per iteration. The Appendix C program required less time per iteration, but the decrease per iteration was not as great. However both programs converged to almost identical costs given equal time. For example, with the weighting P set equal to .1 in each program, the program in Appendix C converged to a cost value of $.470 \times 10^{-2}$ while the second program converged to a cost value of .468 times 10^{-2}.

Convergence Criteria

The value of the cost, the gradient and the Hamiltonian function can be used to check for convergence of the numerical problem. For the results of this study, the values of cost and gradient were checked as well as the values of X2 and X4. In checking the dynamics and the model of Figure 1, it can be seen that if the weighting factor P is zero, the value of the relative velocity X2 between M1 and M2 is equal and opposite in sign to X4, the velocity of M2 relative to the fixed reference. This must be so for the airplane M1 to ride perfectly level over a

bump or depression. For the weighting factor P equal to zero, these values agreed in magnitude to within three and four significant figures. Finally the gradient values for convergence were in the range of 10^{-2} for the discrete time points and the change in cost value between iterations for convergence was set at less than .5 percent. If the computer program was left to iterate after the above criteria had been met, no significant improvement in cost was gained.

VI. Conclusion

The response of the active landing gear system to both the (1-cos wt) input and the decaying exponential input shows a marked improvement in vibration isolation performance over the passive system. To achieve the active system, the actuator is selected and its maximum values of force output and response rate are noted. With these values, the weighting factor F in the cost criterion is chosen such that the system response requires less than or the equivalent of the maximum force and response limitations of the actuator. The designer can then weigh the vibration isolation performance to be gained by the active landing gear system.

The study provides "open loop" solutions for U(t) for both kinds of inputs; however, the solution for U(t) with the decaying exponential has the advantage since the cost function is the integral square criterion with the final time t_f approaching infinity. With this criteria, a constant coefficient feedback controller can be obtained from the open loop controller U(t) (Ref 10,91).

VII. Recommendations

During the course of this study, a number of investigations for further study have been suggested. The following is a list of possible experiments that might prove valuable for the active control landing gear.

1. The rattle space could be constrained and made part of the cost function along with the control force U(t).
2. Nonlinear passive elements could be used to more closely simulate present landing gear suspensions.
3. Improve the dynamic model of this study by providing for wheel diameter size effects and permitting wheel hop for very rough landing surfaces.
4. Investigate model response with various values of Ks and Cs.

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Appendix A

Detailed Problem Formulation

From equation 1 the absolute acceleration $\ddot{Z}$ can be expressed as

$$\ddot{Z} = \frac{F_s(t)}{M_1} - \frac{K_s}{M_1} X_1 - \frac{C_s}{M_1} \dot{X}_1 - g \qquad (14)$$

Setting $U(t) = Fs(t)/F1$ and since $M1 = F1/g$ equation 14 can be written as

$$\ddot{Z} = g\,u(t) - g\frac{K_s}{F_1} X_1 - g\frac{C_s}{F_1} \dot{X}_1 - g \qquad (15)$$

For the minimization of $\ddot{Z}$, it is not necessary to carry g, thus equation 4 in state space notation becomes

$$J(u) = \int_0^{t_f} \left[\left(u(t) - Q_1 X_1 - Q_2 X_2 - 1 \right)^2 + P\,u^2(t) \right] dt \qquad (16)$$

with t_f equal to the period of the cosine function for the (1- cos wt) input and equal to infinity for the decaying exponential input.

Now, in order to evaluate equation 16 in the same integration scheme as equations 6-9, a new state variable

is introduced such that

$$\dot{X}_5 = \left[(u(t) - Q_1 X_1 - Q_2 X_2 - 1)^2 + P u^2(t) \right] \tag{17}$$

where

$$X_5(0) = 0$$

Then equation 16 becomes

$$J(u) = X_5(t_f) \tag{18}$$

and the system differential equations are

$$\begin{aligned}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= (1+R) g u(t) - Q_3 X_1 - Q_4 X_2 \\
&\quad - Q_5 (F(t) - X_3) \\
\dot{X}_3 &= X_4 \\
\dot{X}_4 &= -R g u(t) + Q_6 X_1 + Q_7 X_2 \\
&\quad + Q_5 (F(t) - X_3) - g \\
\dot{X}_5 &= (u(t) - Q_1 X_1 - Q_2 X_2 - 1)^2 + P u^2(t)
\end{aligned} \tag{19}$$

with the initial conditions

$$
\begin{aligned}
X_1(0) &= -F_1/K_s \\
X_2(0) &= 0 \\
X_3(0) &= -(F_1+F_2)/K_t \\
X_4(0) &= 0 \\
X_5(0) &= 0
\end{aligned}
\tag{20}
$$

As a result of the above equations, the Hamiltonian is

$$
\begin{aligned}
H = \; & P_1 X_2 \\
& + P_2 \left[(1+K) g u(t) - Q_3 X_1 \right. \\
& \qquad \left. - Q_4 X_2 - Q_5 (F(t) - X_3) \right] \\
& + P_3 X_4 \\
& + P_4 \left[-R g u(t) + Q_6 X_1 + Q_7 X_2 \right. \\
& \qquad \left. + Q_5 (F(t) - X_3) - g \right] \\
& + P_5 \left[(u(t) - Q_1 X_1 - Q_2 X_2 - 1)^2 \right. \\
& \qquad \left. + P u^2(t) \right]
\end{aligned}
\tag{21}
$$

Then from equation 11, the adjoint equations become

$$\dot{P}_1 = Q_3 P_2 - Q_6 P_4$$

$$+ 2 Q_1 (u(t) - Q_1 X_1 - Q_2 X_2 - 1)$$

$$\dot{P}_2 = - P_1 + Q_4 P_2 - Q_7 P_4$$

$$+ 2 Q_2 (u(t) - Q_1 X_1 - Q_2 X_2 - 1)$$

$$\dot{P}_3 = Q_5 (P_4 - P_2) \tag{22}$$

$$\dot{P}_4 = - P_3$$

$$\dot{P}_5 = 0$$

and from equation 12, the transversality conditions are

$$P_1(t_f) = 0$$

$$P_2(t_f) = 0$$

$$P_3(t_f) = 0 \tag{23}$$

$$P_4(t_f) = 0$$

$$P_5(t_f) = 1$$

$\dot{P}_5$ and $P_5(t_f)$ of equations 28 and 29 respectively can be solved by inspection to give $P_5(t) = 1$. Finally the gradient of equation 13 becomes

$$\frac{dH}{du} = (1+R)g P_2 - R g P_4$$
$$+ 2 \left(u(t) - Q_1 X_1 - Q_2 X_2 - 1 \right)$$
$$+ 2 P \dot{u}(t) \qquad (2[illegible])$$

Appendix B

Derivation of Input F(t) From Spectral Density

The effects of runway uneveness can be represented by a stationary broad band random process (Ref 7). Referring to Figure 3, runway height is commonly described in terms of power spectral density. The spacial frequency Ω (rad/ft) can be transformed to the time frequency ω (rad/sec) by noting that the distance based spectrum $\phi_{xx}(\Omega)$ is related to the time based spectrum $\phi_{xx}(\omega)$ by

$$\phi_{xx}(\omega) = \frac{1}{V}\,\phi_{xx}(\Omega) \tag{25}$$

$$\Omega = \frac{\omega}{V} \tag{26}$$

and V is the velocity of the airplane. From Figure 3 the power spectrum can be approximated by the straight line fit

$$\phi_{xx}(\Omega) = \frac{A}{\Omega^2} \tag{27}$$

or using equation 25 and 26

$$\phi_{xx}(\omega) = \frac{AV}{\omega^2} \tag{28}$$

A is a parameter which is equal to 10^{-5} for most runways (Ref 1:47). However equation 27 becomes infinite at the low frequency end where-as elevation spectra must level off due to the finite height of runways. This can be compensated for by modifying to the form:

$$\Phi_{xx}(\Omega) = \frac{A}{\Omega^2 + \left(\frac{2\pi}{\lambda_0}\right)^2} \tag{29}$$

and again converting to the time based spectrum

$$\Phi_{xx}(\omega) = \frac{AV}{\omega^2 + \left(\frac{2\pi V}{\lambda_0}\right)^2} \tag{30}$$

where λ_0 was chosen to be 5×10^4 ft (Ref 1:47).

The above equation represents the power spectral density or the time based spectrum for an ergotic random process. Therefore $\Phi_{xx}(\omega)$ is the fourier transform of the autocorrelation $R_{xx}(\tau)$

$$R_{xx}(\tau) = \frac{1}{j}\int_{-j\infty}^{j\infty} \Phi_{xx}(s)\, e^{s\tau}\, ds \tag{31}$$

$$s = j\omega$$

The objective is to find an F(t) so that one can minimize the mean square value in equation 5. The mean square value of F(t) is $R_{xx}(0)$ or

$$\overline{F^2} = R_{xx}(0) = \frac{1}{j}\int_{j\infty}^{j\infty} \phi_{xx}(S)\, dS \qquad (32)$$

Using Parseval's theorm, the mean square value can also be written

$$\overline{F^2} = \int_{-\infty}^{\infty} F(t)\, dt = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} F(S)F(-S)\, dS \qquad (33)$$

where F(s) is the fourier transform of F(t) (Ref 6).

Now if $\phi_{xx}(S)$ is multiplied by $2\pi/2\pi$ and the denominator term is taken outside the integral in equation 32, then comparing equations 32 and 33

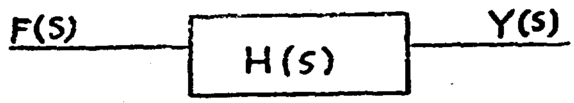
$$F(S)F(-S) = \phi_{xx}(S) = \frac{2\pi AV}{\left(S + \frac{2\pi V}{\lambda_o}\right)\left(-S + \frac{2\pi V}{\lambda_o}\right)} \qquad (34)$$

The time function F(t) is equal to the inverse transform of F(s), or

$$F(t) = F^{-1}\left[\frac{\sqrt{2\pi AV}}{S + \frac{2\pi V}{\lambda_c}}\right] \qquad (35)$$

$$F(t) = u(t)\sqrt{2\pi AV}\; e^{-\frac{2\pi Vt}{\lambda_o}} \qquad (36)$$

U(t) is the unit step function. F(t) represents the equivalent deterministic input derived from the spectral density of the runway. Furthermore, for the linear system,



$$Y(s) = H(s)\,F(s) \tag{37}$$

the random process theory can be extended from equation 32 where

$$\overline{y^2} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} H(s)H(-s)\,\Phi_{xx}(s)\,ds \tag{38}$$

and $\Phi_{xx}(s)$ is defined by equation 34. Then again using Parseval's theorem

$$y^2 = \int y^2(t)\,dt = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} H(s)F(s)H(-s)F(-s)\,dS \tag{39}$$

To summarize the above equations, by using the

decaying exponential input as a measure of randomness of runway height, and by ensuring a linear system, root mean square system output values of force, displacement, velocity etc. can be obtained by computing the square root of the integral square value of the system output force, displacement and velocity respectively.

Appendix C

Digital Computer Program

The IBM 7090 digital computer was used to make the calculations found in previous sections of this study. The program as illustrated in the following pages, is arranged for the (1-cos wt) input. However by not storing all the states and F(t), more of the necessary computer storage space can be gained for use with the decaying exponential input. The list of symbols in the prefatory of this study also applies to the computer program.

```
$IBJOB
$IBFTC MAIN
      COMMON/GOOD/X(5,2001),XX(5,5),DX(5,5),DH(5),
     1G(2,2001),S(2001),U(2,2001),F(2001),KCOUN,GRAV,
     2COST,NNZP,LBO,M,P,Q1,Q2,Q3,Q4,Q5,Q6,Q7,R
      DIMENSION ACC(501)
C   H IS THE DISCRETE TIME INTERVAL
C   IT IS THE NUMBER OF ITERATIONS
C   KCOUN IS THE NUMBER OF DISCRETE TIME POINTS
      FREQ = 8.0
      KCOUN = 401
      RCOUN = KCOUN - 1
      H = 1.0/(FREQ*RCOUN)
      KSTEP = 10
C   GUESS FIRST CONTROL
      DO 16 I = 1,KCOUN
 16   U(1,I) =    0.0
 15   IT = 0
      TEM = 1.E05
      NNZP = 1
      NN = -1
      LBO = -1
      BETA = 0.0
      PII2 = 6.28318
      GRAV = 32.1725
      F1 = 100000.
      F2 = 1000.
      XM1 = F1/GRAV
      XM2 = F2/GRAV
      XCS = 100.0
      XKS = 200000.
      XKT = 1200000.
      P = 0.1
      Q1 = XKS/F1
      Q2 = XCS/F1
      Q3 = XKS*GRAV*(F1+F2)/(F1*F2)
      Q4 = XCS*GRAV*(F1+F2)/(F1*F2)
      Q5 = XKT*GRAV/F2
      Q6 = XKS*GRAV/F2
      Q7 = XCS*GRAV/F2
      R = XM1/XM2
      FS = (SQRT(XKS/XM1))/PII2
      FT = (SQRT(XKT/XM2))/PII2
      WRITE (6,801) FS,FT
 801  FORMAT (2X,2(E15.8,4X))
      TIM = 0.0
      DO 1 I = 1,KCOUN
      S(I) = 0.0
      F(I) = (1.0 - COS(PII2*FREQ*TIM))*0.10416667
      TIM = TIM + H
      DO 1 J = 1,5
 1    X(J,I) = 0.0
```

```
C   INITIAL CONDITIONS
      X(3,1) = -(F1+F2)/XKT
      X(1,1) = -F1/XKS
 4    M = 1
      XX(1,1) = X(1,1)
      XX(2,1) = X(2,1)
      XX(3,1) = X(3,1)
      XX(4,1) = X(4,1)
      XX(5,1) = X(5,1)
C   INTEGRATE  STATE  EQUATIONS  FORWARD
      CALL DER1 (H,1)
      DO 2 I = 2,KCOUN
      M = I
      CALL RUNGE (H,1,5)
      DO 2 J = 1,5
      X(J,I) = XX(J,2)
      DX(J,1) = DX(J,2)
 2    XX(J,1) = XX(J,2)
C INTEGRATE COSTATE EQUATIONS BACKWARDS
      M = KCOUN
      H = -H
C   FINAL CONDITIONS ON COSTATES
      XX(1,1) = 0.0
      XX(2,1) = 0.0
      XX(3,1) = 0.0
      XX(4,1) = 0.0
      CALL DER2  (H,1)
      DO 3 I = 2,KCOUN
      CALL RUNGE (H,1,4)
      DO 3 J = 1,4
      DX(J,1) = DX(J,2)
 3    XX(J,1) = XX(J,2)
      H = -H
C   COMPUTE  COST
      COST = X(5,KCOUN)
      IF ( NNZP ) 5,5,6
C   ALPHA  SEARCH  AND  NEW  CONTROL
 5    CALL ALPHA (H,KOK)
      GO TO 4
 6    CONTINUE
      DO 26 I = 1,KCOUN
 26   ACC(I) = U(1,I) - Q1*X(1,I) - Q2*X(2,I) - 1.0
      WRITE (6,200) IT,FREQ
 200  FORMAT (//, 5X, 12HITERATION = , I6,10X,
     112HFREQUENCY = ,E15.8,//)
      IT = IT + 1
      WRITE (6,201)
 201  FORMAT(5X,4HTIME,12X,8HGRADIENT,11X,9HX1(STATE),
     19X,9HX3(STATE)
     2,9X,11HU1(CONTROL),9X,9HX2(STATE),9X,9HX4(STATE))
      TIM = 0.0
```

```
      TSTEP = KSTEP
      DO 203 I = 1,KCOUN , KSTEP
      WRITE(6,202) TIM , G(1,I),X(1,I),X(3,I),U(1,I),
     1X(2,I),X(4,I)
 202  FORMAT (7(E15.8,4X))
 203  TIM = TIM + H*TSTEP
      WRITE  (6,503)
 503  FORMAT (/////)
      WRITE (6,204) COST
 204  FORMAT(//,5X,6HCOST = , E15.8 )
C  COMPUTE SUM OF ABSOLUTE
C  VALUE OF GRADIENT POINTS
      DIFF = 0.
      DO 7 I = 1,KCOUN
      DEV  = ABS (G(1,I))
 7    DIFF = DIFF+  DEV
      WRITE (6,205) DIFF
 205  FORMAT (14HSUM OF DIFF. =,E15.8,//)
C  CHECK COST AND SUM OF ABSOLUTE
C  VALUE OF GRADIENT POINTS
      IF (IT .LT. 5) GO TO 706
      IF ( DIFF  - 0.5       )  701, 701 , 702
 702  IF (ABS((TEM-COST)/COST) - .005) 701,701,706
 706  TEM = COST
      IF (NN) 10,10,8
C  CALCULATE  BETA
 8    PRO2 = 0.0
      PRO4 = 0.0
      DO 9  I = 1,KCOUN
      PRO1 =  G(1,I) **2
      PRO2 = PRO2 + PRO1
      PRO3 = G(2,I)**2
 9    PRO4 = PRO4 + PRO3
      BETA = PRO2/PRO4
C  STORE CONTROL AND GRADIENT
C  COMPUTE NEW SEARCH DIRECTION
 10   DO 11 I = 1,KCOUN
      U(2,I) = U(1,I)
      G(2,I) = G(1,I)
 11   S(I) = -G(2,I) + BETA*S(I)
      NN = 1
      NNZP = -1
      GO TO 4
 701  WRITE (6,206)
 206  FORMAT(10H**********, 21HLAST TRAJ. IS OPTIMAL,
     110H**********)
      WRITE (7,811) ( U(1,I), I = 1,KCOUN)
      WRITE (7,811) ( ACC(I), I = 1,KCOUN)
 811  FORMAT (5E15.8)
      IF (FREQ-5.0) 31,31,32
 31   IF (FREQ-1.5) 33,33,34
```

```
 33    IF (FREQ-0.6) 37,37,36
 32    FREQ = FREQ /2.0
       GO TO 35
 34    FREQ = FREQ - 1.0
       GO TO 35
 36    FREQ = FREQ/2.0
 35    H = 1.0/(FREQ*RCOUN)
       GO TO 15
 37    CONTINUE
       STOP
       END
$IBFTC SUBA
       SUBROUTINE ALPHA (H,KOK)
       COMMON/GOOD/X(5,2001),XX(5,5),DX(5,5),DH(5),
      1G(2,2001),S(2001),U(2,2001),F(2001),KCOUN,GRAV,
      2COST,NNZP,LBO,M,P,Q1,Q2,Q3,Q4,Q5,Q6,Q7,R
       DIMENSION A(60),COSS(60),PRO4(60)
 30    IF ( LBO) 10, 10, 20
C  ALPHA  COMPUTATION
 10    J = 1
       AAA = 2.0
C  MAKE FIRST GUESS ON ALPHA
       AA = 0.01
       A(J) = AA
C  UPDATE CONTROL, RETURN AND INTEGRATE
       DO 1 I = 2,KCOUN
 1     U(1,I) = U(2,I) + A(J)*S(I)
       NNN = 0
       LBO = 1
       LL = -1
       RETURN
 20    PRO4(J) = 0.0
       COSS(J) = COST
       NNN = NNN + 1
       IF (NNN-50 ) 23,24,24
C  COMPUTE DIRECTIONAL DERIVATIVE
C  OR INTERPRODUCT G*S
 23    DO 2 I = 1,KCOUN
       PRO1 =  G(1,I) * S(I)
 2     PRO4(J) = PRO4(J) + PRO1
       WRITE (6,801) PRO4(J),A(J),COSS(J)
 801   FORMAT (2X,10HPRO4(J) = ,E15.8,4X,7HA(J) = ,E15.8,
      14X,7HCOSS = ,E15.8)
C  CHECK SIGN OF INTERPRODUCT
       IF (PRO4(J)) 4,24,6
 4     LL = 1
 14    J = J + 1
       A(J) = A(J-1)*AAA
       DO 5 I = 2,KCOUN
 5     U(1,I) = U(2,I) + A(J)*S(I)
       RETURN
```

```
 6     IF(LL .EQ. -1) GO TO 9
C   CUBIC  INTERPOLATE  FOR  ALPHA
 7     Z = 3.0*(COSS(J-1)-COSS(J))/(A(J)-A(J-1))+PRO4(J)
      1+PRO4(J-1)
       W = SQRT(Z**2-PRO4(J)*PRO4(J-1))
       AA = A(J)-(PRO4(J)+W-Z)*(A(J)-A(J-1))/(PRO4(J)-
      1PRO4(J-1) + 2.*W)
 24    DO 8 I = 2,KCOUN
 8     U(1,I) = U(2,I) +  AA *S(I)
       LBO = -1
       NNZP = 20
       WRITE (6,802) AA
 802   FORMAT (2X,5HAA = ,E15.8,//)
       RETURN
C   IF FIRST GUESS ON ALPHA GIVES POSITIVE
C   G*S, THEN REDUCE ALPHA GUESS
 9     A(J) = .1 *A(J)
       DO 11 I = 2,KCOUN
 11    U(1,I) = U(2,I) + A(J)*S(I)
       RETURN
       END
$IBFTC SUPR
       SUBROUTINE DER1 (H,I)
       COMMON/GOOD/X(5,2001),XX(5,5),DX(5,5),DH(5),
      1G(2,2001),S(2001),U(2,2001),F(2001),KCOUN,GRAV,
      2COST,NNZP,LBO,M,P,Q1,Q2,Q3,Q4,Q5,Q6,Q7,R
       DX(1,I) = XX(2,I)
       DX(2,I) = (1.+R)*GRAV*U(1,M) - Q3*XX(1,I) -
      1Q4*XX(2,I) - Q5*(F(M)-XX(3,I))
       DX(3,I) = XX(4,I)
       DX(4,I) = -R*GRAV*U(1,M) + Q6*XX(1,I) + Q7*XX(2,I)
      1+Q5*(F(M)-XX(3,I)) - GRAV
       DX(5,I) = (U(1,M) - Q1*XX(1,I) - Q2*XX(2,I) - 1.0 )
      1**2 + P*U(1,M)**2
       RETURN
       END
$IBFTC SUBC
       SUBROUTINE DER2 (H,I)
       COMMON/GOOD/X(5,2001),XX(5,5),DX(5,5),DH(5),
      1G(2,2001),S(2001),U(2,2001),F(2001),KCOUN,GRAV,
      2COST,NNZP,LBO,M,P,Q1,Q2,Q3,Q4,Q5,Q6,Q7,R
C   COMPUTE GRADIENT
       G(1,M) = (1.+R)*GRAV*XX(2,I)- R*GRAV*XX(4,I)+
      1 2.*(U(1,M) - Q1*X(1,M) - Q2*X(2,M) - 1.0)
      2+ 2.*P*U(1,M)
       DX(1,I) = Q3*XX(2,I) - Q6*XX(4,I) +
      1 2.*Q1*(U(1,M) - Q1*X(1,M) - Q2*X(2,M) - 1.0)
       DX(2,I) = -XX(1,I) + Q4*XX(2,I) - Q7*XX(4,I) +
      1 2.*Q2*(U(1,M) - Q1*X(1,M) - Q2*X(2,M) - 1.0)
       DX(3,I) = Q5*(XX(4,I) - XX(2,I))
       DX(4,I) = -XX(3,I)
```

```
      RETURN
       END
$IBFTC SUPD
      SUBROUTINE RUNGE (H,N,IL)
      COMMON/GOOD/X(5,2001),XX(5,5),DX(5,5),DH(5),
     1G(2,2001),S(2001),U(2,2001),F(2001),KCOUN,GRAV,
     2COST,NNZP,LPO,M,P,Q1,Q2,Q3,Q4,Q5,Q6,Q7,R
C   4TH ORDER RUNGE-KUTTA INTEGRATION
      N2 = N + 1
      N4 = N + 3
      DH(N2) =  H/2.0
      DH(N+2)= DH(N2)
      DH(N4) = H
      DO 201 J = N2, N4
      DO 101 K = 1,IL
  101 XX(K,J)=XX(K,N)+DH(J)*DX(K,J-1)
      IF (H) 2,2,1
    1 CALL DER1(H,J)
      GO TO 201
    2 CALL DER2(H,J)
  201 CONTINUE
      DO 401 K = 1,IL
  401 XX(K,N2)=XX(K,N)+H*(DX(K,N)+2.*DX(K,N2)+2.*DX(K,N+2)
     1+DX(K,N4))/6.0
      IF (H) 4,4,3
    3 M = M + 1
      CALL DER1(H,N2)
      GO TO 5
    4 M = M - 1
      CALL DER2(H,N2)
    5 CONTINUE
      RETURN
      END
$IBLDR FILES                    18 DEC 69
$FILE  FILES    -UNIT07-,PP1,READY,OUTPUT,BCD,BLK=14
$FDICT FILES
*4 P7G02-    *(-PPPPPP
$TEXT  FILES
*4 =4H*           I
$CDICT FILES
*5 .*( 3  1-*)P 3 XD-
$DKEND FILES
$EOF
```

VITA

Ronald A. De Yoe was born in Albany, New York on 23 May 1940. He graduated from Colonie Central High School in 1958 and enlisted in the United States Air Force the same year. While serving in the Air Force, he received training as a radio repairman and worked as a radio technician before applying for the Airman Education and commissioning Program (AECP) 1965. Under the AECP program, he attended the University of Oklahoma where he received the degree of Bachelor of Science in Electrical Engineering in 1968 and was elected to Eta Kappa Nu, and Tau Beta Pi. He was commissioned following Officers Training School conducted at Lackland Air Force Base and was directly assigned to the Air Force Institute of Technology in September 1968.

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